Advanced Micro II - Problem set 1
due date: April, 1st

Problem 1 (1p) *Consider the monopolist facing demand $D(p)$ and constant marginal costs c, solving:*

$$\max_{p\in[c,\infty)}(p-c)D(p).$$

State the weakest conditions on D, such that % margin for an optimal price $\hat{m}=\frac{p-c}{p}$ is weakly increasing / decreasing in c.

Problem 2 (1p) *Consider a Cournot duopoly with homogenous product, where: $\pi_i(q_i,q_j)=q_iP(q_i+q_j)-C(q_i)$, where C is the total costs function, and P is inverse demand. State conditions on P and C, such that it is a submodular game, i.e. BR-ses are strong set order decreasing.*

Problem 3 (2p) *Consider Cournot model with homogenous product and n firms, where the profit is as above. Let $2,3,\ldots,n$ firms produce y each. We can rewrite the optimization problem of firm 1 as choosing the total supply z, i.e. if 1 produces $z-(n-1)y$ then:*

$$\Pi(z,y)=(z-(n-1)y)P(z)-C(z-(n-1)y).$$

- *State conditions on P and C, such that Π has ID in (z,y).*
- *How the above argument can be used to prove existence of a symmetric NE of Cournot game using Tarski fixed point theorem? Hint: Read: Amir, Laibson (2000).*

Problem 4 (3p) *Let C be a subset of R^l, and T a subset of R. Consider function $F:R^l\times T\to R$, for which $F(x,t)=\bar{F}(x)+f(x_i,t)$, where $f:R\times T\to R$ is supermodular. Let $x''\in\arg\max_{x\in C}F(x,t'')$ and $x'\in\arg\max_{x\in C}F(x,t')$ for any $t''>t'$. Show that, if $x_i'>x_i''$ then $x''\in\arg\max_{x\in C}F(x,t')$ and $x'\in\arg\max_{x\in C}F(x,t'')$.*

Problem 5 (3p) *Consider a consumer maximizing utility $U:R^L_{++}\to R$ with $U(x)=v(x_2,x_3,\ldots,x_L)+u(x_1)$ subject to $p\cdot x\leq w$. Assume $p\gg 0$ and $w>0$. Assume u is twice differentiable with $u'(x_1)>0$. Assume for simplicity that solution to the utility maximization problem exists and is unique. Show that the increase of price of good 1 causes the consumer to reduce his expenditure on good 1 if $-x_1u''(x_1)/u'(x_1)<1$. HINT: use the previous problem to answer that.*

Suppose in addition that $v(x_2,x_3,\ldots,x_L)=\sum_{i=2}^{L}u_i(x_i)$ where each u_i is concave. How does the demand for goods $2,3,\ldots$ react to an increase in price of good 1?

Problem 6 (3p) *Let X and S and T be intervals in R. The functions $g:X\times S\to R_{++}$ and $h:S\times T\to R_{++}$ are both log-supermodular. Show that $G:X\times T\to R_{++}$ given by*

$$G(x,t)=\int_S g(x,s)h(s,t)ds$$

is log-supermodular.