Theorem 1 (Minkowski) *Let A, B be two covex sets in $\mathbb{R}^L$ and $A \cap B = \emptyset$. Then there exists $p \in \mathbb{R}^L$, $p \neq 0$ and a number $z \in \mathbb{R}$ such that, $p \cdot x \geq z \geq p \cdot y$ for any $x \in A, y \in B$.*

Consider a maximization problem $\max_{x \in X} f(x)$ constrained by $g_k(x) \leq w_k$ for all $k = 1, \ldots, K$. We say that $x \in X$ is feasible, if $g_k(x) \leq w_k$ for all $k = 1, \ldots, K$.

Theorem 2 (Kuhn-Tucker) *Let $X \subset \mathbb{R}^N$ be convex, $f : X \to (-\infty, \infty)$ concave and $g_k : X \to (-\infty, \infty)$ convex. Let x^* be feasible and there exists nonnegative numbers $\lambda_1, \ldots, \lambda_K$ such that:*

1. *for any k $\lambda_k = 0$, where $g_k(x^*) < w_k$,*
2. *x^* solves: $\max_{x \in X}[f(x) - \sum_{k=1}^{K} \lambda_k g_k(x)]$,*

then x^* solves the maximization problem.

Let x^* solve the maximization problem and there exists $\underline{x} \in X$, such that $g_k(\underline{x}) < w_k$ for any k. Then there exist non-negative numbers $\lambda_1, \ldots, \lambda_k$ such that 1 and 2 hold.

Theorem 3 (Brouwer) *Let $A \subset \mathbb{R}^L$ be non-empty, compact and convex set and function $f : A \to A$ continuous. Then there exists $x^* \in A$, such that $x^* = f(x^*)$.*

Theorem 4 (On local invertability of a differentiable function) *Let $f : U \to \mathbb{R}^L$, where $U \in \mathbb{R}^n$ is open, be continuously differentiable on $B_r(x_0) \subset U$ and $\det f'(x_0) \neq 0$. Then there exists a neighbourhood $O = B_\epsilon(x_0)$ ($\epsilon < r$), such that function $f|_O : O \to V$, (gdzie $f(O) = V$) is invertible.*

Definition 1 (Upper-hemi continuous correspondence) *A correspondence $f : A \to a^Y$, where $A \subset \mathbb{R}^L$, and $Y \subset \mathbb{R}^K$ is closed is called upper-hemi continuous, if it has a closed graph and images of compact sets are bounded.*

Definition 2 (Lower-hemi continuous correspondence) *A correspondence $f : A \to 2^Y$, where $A \subset \mathbb{R}^L$, and $Y \subset \mathbb{R}^K$ is compact, is called lower-hemi continuous, if for any sequence (elements of A) $x_m \to x \in A$ and any $y \in f(x)$, there exists a sequence $y_m \to y$ and number M such that $y_m \in f(x_m)$ for $m > M$.*

Theorem 5 (Berge maximum theorem) *Let $f : Y \to \mathbb{R}$ be continuous, and correspondence $\Gamma : X \to 2^Y$ continuous[1]. If $\Gamma(x) \neq \emptyset$, then function $M(x) = \max\{f(y) : y \in \Gamma(x)\}$ is continuous on X, and correspondence $\Phi(x) = \{y \in \Gamma(x) : f(y) = M(x)\}$ is upper-hemi continuous.*

[1] I.e. both upper- and lower-hemicontinuous.

Theorem 6 (Kakutani) *Let $A \subset \mathbb{R}^L$ be a nonempty, compact and convex set and a correspondence $f : A \to 2^A$ upper-hemi continuous. If $f(x)$ is non-empty and convex for every $x \in A$, then there exists $x^* \in A$ such that $x^* \in f(x^*)$.*

Theorem 7 (Shapley-Folkman) *Let $x \in con(\sum_{i=1}^{I} A_i)$, where $(\forall i) A_i \subseteq \mathbb{R}^L$. Then, there exists $(a_i)_{i=1}^{I}$, such that $x = \sum_{i=1}^{I} a_i$, and $(\forall i) a_i \in con(A_i)$ with $a_i \in A_i$ for all but L indexes i.*