Université Paris 1 Panthéon–Sorbonne | December 3, 2014 – 11:00 A.M.
Microeconomics 1 – Master M1 MAEF/MMEF/QEM | Elena L. del Mercato

Exam (1 hour and 40 minutes)

Read and think before you write, and try to be both concise and precise.

Exercise 1 (30 minutes). Let L be the finite number of commodities and let $\mathcal{E} = (u_i, e_i)_{i=1,\dots,m}$ be a pure exchange economy with m consumers.

1. State the theorem on the existence of a competitive equilibrium.
2. State the First Theorem of Welfare Economics.
3. Now, consider a specific pure exchange economy with $L = 2$ commodities and $m = 2$ consumers. The individual utility functions are given by

$$u_1(x_1^1, x_1^2) = x_1^1 \quad \text{and} \quad u_2(x_2^1, x_2^2) = x_2^2$$

$e_1 = (1, 1)$ and $e_2 = (2, 1)$ are the individual initial endowments.

   (a) Take a point in the interior of the Edgeworth box, and for both consumers draw the indifference curves and the upper contour sets associated with this point.
   (b) Write the definition of a Pareto optimal allocation for this specific economy.
   (c) Using the Edgeworth box and the definition of a Pareto optimal allocation, determine geometrically the set of Pareto optimal allocations.
   (d) Show that for every consumer $i = 1, 2$, the utility function u_i is quasi-concave and monotonic.
   (e) Can we apply the theorem on the existence of a competitive equilibrium? Can we apply the First Theorem of Welfare Economics? Deduce what is the equilibrium allocation.
   (f) Using the equilibrium allocation and the properties of a competitive equilibrium, compute the equilibrium price.

Exercise 2 (50 minutes).

1. Let L be the finite number of commodities and let $\mathcal{E} = (Y, (u_i, e_i)_{i=1,\dots,m})$ be a production economy with one firm and m consumers. Assume that the production set Y is represented by a transformation function t. State the proposition on the characterization of Pareto optimal allocations in terms of first order conditions.
2. Now, consider a specific production economy with $L = 3$ commodities, one firm and $m = 2$ consumers. The firm produces two outputs, namely commodities 1 and 2, using commodity 3 as an input. The generic production plan of the firm is denoted by $y = (y^1, y^2, y^3)$ and the production set of the firm is given by

$$Y = \left\{ y = (y^1, y^2, y^3) \in \mathbb{R}^3 \colon y^1 + \beta y^2 \leq 2\sqrt{-y^3} \ \text{ and } y^3 \leq 0 \right\}$$

So, the production set Y is represented by the transformation function

$$t(y^1, y^2, y^3) = y^1 + \beta y^2 - 2\sqrt{-y^3} \text{ with } y^3 \leq 0$$

where the parameter β represents the marginal rate of transformation of output 2 for output 1, we assume that $\beta > 1$.

The two consumers have the same preferences and the same initial endowments. For every consumer $i = 1, 2$, the generic consumption bundle of consumer i is denoted by $x_i = (x_i^1, x_i^2, x_i^3) \in \mathbb{R}^3_+$, the preferences of consumer i are represented by the utility function $u_i(x_i^1, x_i^2, x_i^3) = x_i^1 + \sqrt{x_i^2} + \sqrt{x_i^3}$ and $e_i = (0, 0, 12)$ is his initial endowment.

(a) Determine the set of all Pareto optimal allocations with strictly positive consumption bundles.

(b) Determine the specific Pareto optimal allocation $(\overline{x}_1, \overline{x}_2, \overline{y})$ which guarantees the same consumption in commodity 1 for all the consumers. Compute the supporting price p^* of this specific Pareto optimal allocation $(\overline{x}_1, \overline{x}_2, \overline{y})$ (the price of commodity 1 is normalized to 1).

From now on, we assume that <u>consumer 1 is the only owner of the firm</u>.

(c) Write the definition of a competitive equilibrium for this specific economy.

(d) Consider $(p^*, (\overline{x}_1, \overline{x}_2, \overline{y}))$ determined in question (b). Show that $(p^*, (\overline{x}_1, \overline{x}_2, \overline{y}))$ is not a competitive equilibrium of this economy.

(e) Determine a redistribution of the initial endowments of commodity 3 for which $(p^*, (\overline{x}_1, \overline{x}_2, \overline{y}))$ can be achieved as a competitive equilibrium.

Exercise 3 (20 minutes). Consider a pure exchange economy with $L = 2$ commodities and $m = 2$ consumers. For every consumer $i = 1, 2$, $e_i = (e_i^1, e_i^2) \in \mathbb{R}^2_{++}$ is the initial endowment of consumer i, and the preferences of consumer i are represented by the Log-Linear utility function

$$u_i(x_i^1, x_i^2) = a_i^1 \ln x_i^1 + a_i^2 \ln x_i^2$$

where $a_i = (a_i^1, a_i^2) \gg 0$ is a parameter such that $a_i^1 + a_i^2 = 1$.

1. Quickly verify that u_i is differentiable and concave on $\mathbb{R}^2_{++}$. Compute the individual demand of consumer i at the price $p = (p^1, p^2) \in \mathbb{R}^2_{++}$.

2. We remind that a competitive equilibrium is called a *no-trade* equilibrium if, at equilibrium, for all consumer $i = 1, 2$, the individual demand of consumer i is equal to his initial endowment. Determine the property that the parameters a_i must satisfy at a *no-trade* equilibrium.