$L = 2$ is the number of commodities and $\mathbb{R}^2_+$ is the consumption set of the consumer.

1. (Linear preferences) For all $x = (x_1, x_2) \in \mathbb{R}^2_+$ and $\overline{x} = (\overline{x}_1, \overline{x}_2) \in \mathbb{R}^2_+$,

$$x \succsim \overline{x} \Longleftrightarrow ax_1 + bx_2 \geq a\overline{x}_1 + b\overline{x}_2$$

   with $a > 0$ and $b > 0$. For every $\overline{x} \in \mathbb{R}^2_+$, determine and draw the indifference curve $I(\overline{x})$ and the upper contour set $U(\overline{x})$.

2. (Leontief preferences). For all $x = (x_1, x_2) \in \mathbb{R}^2_+$ and $\overline{x} = (\overline{x}_1, \overline{x}_2) \in \mathbb{R}^2_+$,

$$x \succsim \overline{x} \Longleftrightarrow \min\{x_1, x_2\} \geq \min\{\overline{x}_1, \overline{x}_2\}$$

   For every $\overline{x} \in \mathbb{R}^2_+$, determine and draw the indifference curve $I(\overline{x})$ and the upper contour set $U(\overline{x})$.

3. (Lexicographic preferences). For all $x = (x_1, x_2) \in \mathbb{R}^2_+$ and $\overline{x} = (\overline{x}_1, \overline{x}_2) \in \mathbb{R}^2_+$,

   $x \succsim \overline{x} \Longleftrightarrow$ " $x_1 > \overline{x}_1$ " or " $x_1 = \overline{x}_1$ and $x_2 \geq \overline{x}_2$ "
   (a) For every $\overline{x} \in \mathbb{R}^2_+$, determine and draw the upper contour set $U(\overline{x})$.
   (b) Show that for every $\overline{x} \in \mathbb{R}^2_+$, the indifference set $I(\overline{x})$ is a singleton.

4. Reconsider linear preferences with $a > 0$ and $b > 0$. Show that this preference relation is continuous, convex, strictly monotone, but not strictly convex.

5. Reconsider Leontief preferences. Show that this preference relation is continuous, convex, monotone, but it is not strictly convex and it is not strictly monotone.

6. Reconsider Lexicographic preferences. Show that this preference relation is strictly monotone and strictly convex, but not continuous.

7. Let $p = (p_1, p_2) \gg 0$ be a price system and let $w > 0$ be the wealth of the consumer. Determine **graphically** the demand of the consumer for the three following cases.
   (a) Linear preferences.
   (b) Leontief preferences.
   (c) Lexicographic preferences.

8. (Cobb-Douglas utility function). For all $x = (x_1, x_2) \in \mathbb{R}^2_+$,

$$u(x_1, x_2) = (x_1)^\alpha (x_2)^{1-\alpha} \text{ with } 0 < \alpha < 1$$

   (a) For every $\overline{x} \in \mathbb{R}^2_+$, determine and draw the indifference curve $I(\overline{x})$ and the upper contour set $U(\overline{x})$.
   (b) Determine the following properties of u: continuity, differentiability, (strictly) increasing, (strictly) (quasi-)concavity.