Cost Minimization

1. $L = 3$ is the number of commodities. The firm produces y using commodities 1 and 2 as inputs. The cost function is given by

$$C(w_1, w_2, y) = 2(y)^2 (w_1)^{\frac{2}{3}} (w_2)^{\frac{1}{3}}$$

Compute the supply and the profit function of the firm.

2. Let L be the number of commodities. A firm produces commodity y using the other $L-1$ commodities as inputs. $z := (z_1, ..., z_l, ..., z_{L-1}) \in \mathbb{R}_+^{L-1}$ denotes a generic bundle of inputs. Show that if the production function $f : \mathbb{R}^{L-1} \longrightarrow \mathbb{R}_+$ is concave, then the the cost function $C(w, y)$ is convex with respect to the output level $y \in \mathbb{R}_+$.

3. $L = 3$ is the number of commodities. The production function is given by

$$f(z_1, z_2) = (z_1)^\alpha (z_2)^\beta \text{ with } \alpha > 0,\ \beta > 0,\ z_1 \geq 0 \text{ and } z_2 \geq 0.$$

Use the demand of inputs and the cost function already determined in a previous exercise, determine the supply of the firm.

Equilibria and Optimality

4. Consider a pure exchange economy with $L = 2$ commodities and $m = 2$ consumers. The individual utility functions are linear functions given by

$$u_1(x_{11}, x_{12}) = x_{11} + x_{12} \quad \text{and} \quad u_2(x_{21}, x_{22}) = ax_{21} + bx_{22}$$

$e_1 = (2, 2)$ and $e_2 = (2, 1)$ are the initial endowments.

(a) Draw the Edgeworth box associated with this economy and represent the feasible allocation (e_1, e_2).

(b) Consider the point $(1, \frac{5}{2})$ and determine the individual consumptions associated with this point.

(c) Write the definition of a competitive equilibrium for this specific pure exchange economy.

(d) Take $a = 1$ and $b = 1$,

1. Represent the indifference curves of both consumers in the Edgeworth box.
2. Using the definition and the properties of competitive equilibria, determine **geometrically** the competitive equilibria of this pure exchange economy.

(e) Take $a = 1$ and $b = 2$, same questions.