1. $L = 2$ is the number of commodities. The firm produces commodity y_2 using commodity z as an input. The production function is given by $f(z) = \alpha\sqrt{z}$ with $\alpha > 0$ and $z \geq 0$.
   (a) Write the transformation function and the profit maximization problem (PMP) of this firm, where $y_1 = -z$ and $y_2 = f(z)$.
   (b) Show that if $\bar{y} = (y_1, y_2)$ belongs to the supply of the firm, then $y_1 < 0$ and $y_2 > 0$.
   (c) Consider the open and convex set $A = \{(y_1, y_2) \in \mathbb{R}^2 \colon y_1 < 0 \text{ and } y_2 > 0\}$. Write the first order conditions associated with (PMP) on the set A, and determine if these conditions are necessary and/or sufficient to solve (PMP) on the set A.
   (d) Compute the supply and the profit function of the firm.

2. Let L be the finite number of commodities. A firm produces commodity L using the other $L-1$ commodities as inputs. $z := (z_1, ..., z_l, ..., z_{L-1}) \in \mathbb{R}_+^{L-1}$ denotes a generic bundle of inputs. Show that if the production function $f : \mathbb{R}_+^{L-1} \longrightarrow \mathbb{R}_+$ is differentiable and concave on the interior of $\mathbb{R}_+^{L-1}$, then the transformation function defined by

$$t_f(y) := y_L - f(z)$$

   is differentiable and quasi-convex on the open and convex set $A = \{y = (-z, y_L) \in \mathbb{R}^L \colon z \gg 0 \text{ and } y_L > 0\}$ [*Suggestion*: Use the first order characterization of concave and quasi-convex functions].

3. $L = 3$ is the number of commodities. The firm produces commodity 3 using commodities 1 and 2 as inputs. The production function is given by

$$f(z_1, z_2) = (z_1)^\alpha (z_2)^\beta$$

   with $z_1 \geq 0, z_2 \geq 0, \alpha > 0,\ \beta > 0, \alpha + \beta \leq 1$.
   (a) Write the transformation function and the profit maximization problem (PMP) of this firm.
   (b) Consider the open and convex set $A = \{y = (-z_1, -z_2, y_3) \in \mathbb{R}^3 \colon z_1 > 0, z_2 > 0, \text{ and } y_3 > 0\}$. Write the first order conditions associated with (PMP) on the set A, and determine if these conditions are necessary and/or sufficient to solve (PMP) on the set A.
   (c) Compute the supply and the profit function of the firm [*Suggestion*: Distinguish the two cases $\alpha + \beta < 1$ and $\alpha + \beta = 1$].

4. Using the definition of the profit function π, prove that π is a convex function.

5. $L = 2$ is the number of commodities. The firm produces commodity 2 using commodity 1 as an input.
   (a) The production function is $f(z) = \alpha(1 - \exp(-kz))$ with $k > 0$, $\alpha > 0$ and $z \geq 0$.
      - Determine and draw the production set Y determined by the production function f.
      - For every level of output $\overline{y}_2 \geq 0$, determine and draw the following set

$$Y(\overline{y}_2) := \{z \in \mathbb{R} \colon z \geq 0 \text{ and } f(z) \geq \overline{y}_2\}$$

      - Write the cost minimization problem of this firm.
      - Determine the demand of inputs and the cost function of the firm.
   (b) The production function is $f(z) = \alpha\sqrt{z}$ with $\alpha > 0$ and $z \geq 0$, same questions.
   (c) The production function is $f(z) = \alpha z^2 + \beta z$ with $\alpha > 0,\ \beta > 0$ and $z \geq 0$, same questions.

6. $L = 3$ is the number of commodities. The firm produces commodity 3 using commodities 1 and 2 as inputs. The production function is given by

$$f(z_1, z_2) = (z_1)^\alpha (z_2)^\beta \text{ with } \alpha > 0,\ \beta > 0,\ z_1 \geq 0 \text{ and } z_2 \geq 0$$

   with $\alpha + \beta \leq 1$. Determine the demand of inputs and the cost function of the firm [*Suggestion*: Distinguish the two cases $\alpha + \beta < 1$ and $\alpha + \beta = 1$].