



**Optimization.
Midterm Exam
October 19, 2015.**

1. Consider the set

$$A = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq \ln x, 1 \leq x \leq 2\}$$

- (i) Draw the set A , its boundary and its interior. Discuss whether the set A is open, closed, bounded, compact and/or convex. Explain your answer.
- (ii) Argue that the function $f(x, y) = y^2 + (x - 1)^2$ has a maximum and a minimum on A .
- (iii) Using the level sets of $f(x, y)$, find the maximum and minimum of f on A .

2. Let

$$f(x) = \begin{cases} x^a & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$$

- (i) Find the values of a such that $f(x)$ is continuous at zero.
 - (ii) Find the values of a such that $f(x)$ is differentiable at zero.
3. Consider a competitive economy with two goods x and y . A consumer in this economy aims at selecting a bundle (x^*, y^*) yielding the highest level of satisfaction given the prices $p_x > 0$ and $p_y > 0$ of the goods and an income endowment $w > 0$.
- (i) Formulate the problem as a non-linear programming problem
 - (ii) Find all possible solutions
 - (iii) Provide an economic interpretation to the conditions characterizing the solutions.