Please explain your responses carefully

1. A friend of yours is afraid of the number 13. Let the alternatives be $X = \mathbb{N} = \{0, 1, 2, \ldots\}$. Does the following relation $\succsim$ satisfy completeness and transitivity? For any $x, y \in \mathbb{N}$:
   - if $x = 13$, then $x \precsim y$.
   - if $x, y \neq 13$ and $x \geq y$, then $x \succsim y$.

2. A consumer in a two-good economy has a demand function $x^*(p, w)$ that satisfies Walras's law. His demand for the first good satisfies $x_1^*(p, w) = \frac{w}{3p_1}$.
   (a) Derive his demand for the second good. Comment.
   (b) Explain why this demand function is consistent with homothetic preferences.

3. Suppose a one-output technology with a production function f. Show that the cost function $C(w, \overline{y})$ is concave in the factor prices w.

4. Consider a production set $Y = \{(y_1, y_2) \in \mathbb{R}^2 : y_1 \leq 1, y_2 \leq 1, y_1 + y_2 \leq 0\}$ which satisfies free disposal.
   (a) Represent the set Y in a figure.
   (b) Does the technology satisfy nonincreasing returns to scale?
   (c) Find the profit function $\pi : \mathbb{R}^2_{++} \to \mathbb{R}$.

5. Consider a nonempty production set Y and its profit function π. Prove or disprove the following statement:
   For any $\lambda \in [0, 1]$ and any two prices $p, q \in \mathbb{R}^2_{++}$, we have

$$\pi(p) \geq \pi(q) \Rightarrow \pi(p) \geq \pi(\lambda p + (1 - \lambda)q) \geq \pi(q).$$