Université Paris 1 Panthéon–Sorbonne
**Microeconomics 1** – Master M1 MAEF/MMEF/QEM

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**Midterm Exam (1 hour and 45 minutes)**

Read and think before you write, and try to be both concise and precise

**Course Questions (25 minutes).** Let $L$ be the finite number of commodities.

**Consumer Theory**

**1.** Let $\succsim$ be a preference relation on $\mathbb{R}^L_+$. Give the following three definitions: $\succsim$ is monotone on $\mathbb{R}^L_+$, $\succsim$ is continuous on $\mathbb{R}^L_+$, $\succsim$ is convex on $\mathbb{R}^L_+$.

**2.** Give the definition of the demand of a consumer at the price system $p \in \mathbb{R}^L_{++}$ and the wealth $w \geq 0$.

**Producer Theory**

**3.** Give the definition of increasing returns to scale for a production set $Y \subseteq \mathbb{R}^L$.

**4.** Write the first order conditions associated with the profit maximization problem (PMP), and give the assumptions under which these conditions are necessary and sufficient to solve (PMP).

**5.** Write the first order conditions associated with the cost minimization problem (CMP), and give the assumptions under which these conditions are necessary and sufficient to solve (CMP).

**Exercise 1 (20 minutes).** Let $L = 2$ be the number of commodities. The preferences of a consumer are described by the preference relation $\succsim$ defined by

$$\forall\, x = (x^1, x^2) \in \mathbb{R}^2_+ \text{ and } \forall\, \overline{x} = (\overline{x}^1, \overline{x}^2) \in \mathbb{R}^2_+ : \quad x \succsim \overline{x} \iff x^1 \geq \overline{x}^1$$

1. Let $\overline{x} \in \mathbb{R}^2_+$, determine and draw the upper contour set $U(\overline{x})$ and the indifference curve $I(\overline{x})$.
2. Show that $\succsim$ is monotone, continuous, and convex on $\mathbb{R}^2_+$. Provide a utility function which represents $\succsim$.
3. Determine **graphically** the demand of the consumer at the price system $p = (p^1, p^2) \gg 0$ and the wealth $w > 0$.

**Exercise 2 (25 minutes).** Let $L = 3$ be the number of commodities. A firm produces the commodity 3 using the commodities 1 and 2 as inputs. The production function of the firm is given by

$$f(y^1, y^2) = y^1 y^2 \quad \text{with } y^1 \leq 0 \text{ and } y^2 \leq 0$$

1. Write the transformation function and the profit maximization problem (PMP) of this firm.
2. Consider the open and convex set $A = \{y = (y^1, y^2, y^3) \in \mathbb{R}^3 \colon y^1 < 0, y^2 < 0, \text{ and } y^3 > 0\}$. Write the system determined by the first order conditions associated with (PMP) on the set $A$. Compute the solution $\overline{y}$ of this system and the profit associated with $\overline{y}$.

3. Determine if the first order conditions are necessary and/or sufficient to solve (PMP).
4. Show that the production set of this firm exhibits increasing returns to scale. Determine the supply of the firm and compare the result with the results found in the previous questions.

**Exercise 3 (35 minutes).** Let $L = 3$ be the number of commodities. The production function $f$ is the same as that given in **Exercise 2**. $(p^1, p^2) \gg 0$ denotes the system of inputs prices.

1. Write the cost minimization problem (CMP) of this firm.
2. Give the definitions of the demand of inputs $D(p^1, p^2, \overline{y}^3)$ and the cost function $C(p^1, p^2, \overline{y}^3)$ of this firm.
3. Notice that $f$ is differentiable and quasi-concave on the interior of $\mathbb{R}^2_-$. Fix a level of output $\overline{y}^3 > 0$, and compute $D(p^1, p^2, \overline{y}^3)$ using the first order conditions associated with (CMP).
4. Fix the level of output $\overline{y}^3 = 0$.

   **a)** Draw the set $Y(0)$ of all input quantities allowing to produce at least $\overline{y}^3 = 0$.

   **b)** Using the iso-cost lines and the shape of the set $Y(0)$, determine **graphically** $D(p^1, p^2, 0)$.
5. Finally, compute the cost function of the firm as a function of $(p^1, p^2, \overline{y}^3)$ for every $\overline{y}^3 \geq 0$.