Microeconomics 1B – QEM/MMEF | Université Paris 1 Panthéon–Sorbonne
Elena L. del Mercato | 2012-2013

Final Exam (2 hours)

Course Questions. Let L be the finite number of commodities.

1. State the proposition which relates the supply of a firm and the derivatives of the cost function.

2. Give the definition of a competitive equilibrium in a private ownership economy.

Consider a pure exchange economy with m consumers.

3. State the proposition on the first order characterization of competitive equilibria.

4. State the proposition on the characterization of Pareto optimal allocations in terms of marginal rates of substitution.

5. State the First Welfare Theorem.

Exercise 1. Let $L = 2$ be the number of commodities. A firm produces commodity 2 using commodity 1 as an input. The cost function of the firm is given by $C(p^1, y_1^2) = 2(y_1^2)^2 p^1$ with $y_1^2 \geq 0$.

Determine the supply $y(p)$ and the optimal profit $\pi(p)$ for any price system $p = (p^1, p^2) \gg 0$.

Exercise 2. We consider a pure exchange economy with $L = 2$ commodities and $m = 2$ consumers. The total initial endowment is $e = (3, 2)$. The consumers have the same preferences represented by $u_i(x_i^1, x_i^2) = x_i^1 + \sqrt{x_i^2}$ for all $i = 1, 2$.

1. Verify that for all $x_1^1 \in]0, 3[$, $\nabla u_1(x_1^1, 1)$ and $\nabla u_2(3 - x_1^1, 1)$ are equal to the vector $(1, \frac{1}{2})$. Deduce that for all $x_1^1 \in]0, 3[$, the allocation $((x_1^1, 1), (3 - x_1^1, 1))$ is a Pareto optimal allocation.

2. Define the price system $p^* = (1, \frac{1}{2})$, and consider the individual initial endowments $e_1 = (2, 0.5)$ and $e_2 = (1, 1.5)$. Determine $x_1^{*1} \in]0, 3[$ such that $p^* \cdot (x_1^{*1}, 1) = p^* \cdot e_1$. Show that $p^* \cdot (3 - x_1^{*1}, 1) = p^* \cdot e_2$.

3. From the previous questions, deduce that $(p^*, (x_1^{*1}, 1), (3 - x_1^{*1}, 1))$ is a competitive equilibrium of the economy defined in question 2.

4. Let $((x_1^1, x_1^2)(x_2^1, x_2^2)) \gg 0$ be a feasible allocation of this economy such that $x_1^2 + x_2^2 = 2$. Show that if $x_1^2 \neq 1$, then this allocation is not a Pareto optimal allocation.

5. Consider different individual initial endowments $(\widetilde{e}_1, \widetilde{e}_2)$ such that $\widetilde{e}_1 + \widetilde{e}_2 = e$. Let $(\widetilde{p}, \widetilde{x}_1, \widetilde{x}_2)$ be a competitive equilibrium for this economy with $(\widetilde{x}_1, \widetilde{x}_2) \gg 0$. Using the First Welfare Theorem and the previous question, show that $\widetilde{x}_1^2 = \widetilde{x}_2^2 = 1$ and $\widetilde{p}$ is proportional to p^*.