Microeconomics 1B – QEM/MMEF — Université Paris 1 Panthéon–Sorbonne
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Final Exam (2 hours)

Course Questions.

1. Give the definition of a production economy.

2. State the proposition on the characterization of Pareto optimality in terms of first order conditions. What is the underlying maximization problem under constraints that leads to this characterization?

3. Give the definition of a private ownership economy.

4. State the First Welfare Theorem in a private ownership economy.

Exercise 1. We consider a private ownership economy with two commodities, two consumers and one firm. The individual utility functions are given by

$$u_1(x_1^1, x_1^2) = (x_1^1)^{\frac{1}{2}}(x_1^2)^{\frac{1}{2}} \quad \text{and} \quad u_2(x_2^1, x_2^2) = x_2^1 x_2^2$$

The individual endowments are $e_1 = (\frac{5}{3}, \frac{1}{4})$ and $e_2 = (\frac{1}{3}, \frac{3}{4})$. The firm produces commodity 1 using commodity 2 as an input. The production set of the firm is given by

$$Y = \{(y^1, y^2) \in \mathbb{R}^2 \colon y^1 \leq 2\sqrt{-y^2} \text{ and } y^2 \leq 0\}$$

The share of consumer 1 is $\theta \in [0, 1]$. Consumer 2 owns the remaining share.

1. Write the definition of competitive equilibrium for this specific private ownership economy.

From now on, the price of commodity 2 is normalized to 1, i.e. $p^2 = 1$.

2. Compute the supply and the optimal profit of the firm.
3. Compute the demand of the consumers.
4. Compute the unique competitive equilibrium with strictly positive consumptions.
5. The equilibrium allocation is Pareto optimal. Why so?

Exercise 2. We consider a production economy with two commodities, two consumers and two firms. The aggregate endowment is given by $(e^1, e^2) = (10, 27)$. The individual utility functions are given by

$$u_1(x_1^1, x_1^2) = 2\ln x_1^1 + \ln x_1^2 \quad \text{and} \quad u_2(x_2^1, x_2^2) = \frac{2}{3}\ln x_2^1 + \frac{1}{3}\ln x_2^2$$

Both firms produce commodity 1 using commodity 2 as an input. The production sets of the firms are given by

$$Y_1 = \{(y_1^1, y_1^2) \in \mathbb{R}^2 \colon y_1^1 \leq -\frac{1}{2}y_1^2 \text{ and } y_1^2 \leq 0\} \text{ and } Y_2 = \{(y_2^1, y_2^2) \in \mathbb{R}^2 \colon y_2^1 \leq \sqrt{-y_2^2} \text{ and } y_2^2 \leq 0\}$$

1. Write the definition of feasible allocation for this specific production economy.
2. Write the definition of Pareto optimal allocation for this specific production economy.

3. Using the characterization of Pareto optimality in terms of first order conditions (or the characterization in terms of marginal rates of substitution and marginal rates of transformation), show that any Pareto optimal allocation (x_1, x_2, y_1, y_2) with $(x_1, x_2) \gg 0$ is a feasible allocation satisfying the following equations.

$$\frac{x_1^2}{x_1^1} = 1 = \frac{x_2^2}{x_2^1}, \;\; y_1^1 = -\frac{1}{2}y_1^2, \;\; y_2^2 = -1, \;\; y_2^1 = 1$$

4. Using x_2^2 as a parameter, determine the set of all Pareto optimal allocations (x_1, x_2, y_1, y_2) with $(x_1, x_2) \gg 0$.

Exercise 3. Prove the First Welfare Theorem in a private ownership economy.