

Problem 1 (2p, subjective probabilities) Consider an exchange economy with two consumers (and subjective probability) and a single consumption good in two states of the world. Consumer i has a von Neumann-Morgenstern utility: $\pi_i \ln(x_{1,i}) + (1 - \pi_i) \ln(x_{2,i})$. Probabilities are, hence, subjective. Let the initial endowment be: $\omega_1 = (1, 0)$ and $\omega_2 = (0, 1)$. Find

- an Arrow-Debreu competitive equilibrium for this economy. Explain, how equilibrium prices depend on subjective π_i .
- a Radner equilibrium (as analyzed in class, i.e. with 2 dates, and Arrow securities).

Are the equilibrium allocations of goods x^* the same? How about prices p^* ?

Problem 2 (2p, aggregate risk) Consider an exchange economy with two consumers (and objective probability) and a single consumption good in two states of the world. Consumer i has a von Neumann-Morgenstern utility: $\pi \ln(x_{1,i}) + (1 - \pi) \ln(x_{2,i})$. Probabilities are, hence, subjective. Let the initial endowment be: $\omega_1 = (1, 0)$ and $\omega_2 = (0, 2)$. There is an aggregate risk. Find

- an Arrow-Debreu competitive equilibrium for this economy. Explain, how equilibrium prices depend on subjective π_i .
- a Radner equilibrium (as analyzed in class, i.e. with 2 dates, and Arrow securities).

Why, even if $\pi = 0.5$ the equilibrium prices of both goods are not the same? Explain.

Problem 3 (2p, aggregate risk 2) Consider an exchange economy with two consumers and a single consumption good in two states of the world ($s = 1$ or $s = 2$). Consumer i has a utility: $\min_{s=1,2} \{\pi_i(s) \ln(x_{s,i})\}$. Let the initial endowment be: $\omega_1 = (1, 0)$ and $\omega_2 = (0, 2)$. There is an aggregate risk. Find an Arrow-Debreu competitive equilibrium for this economy. Explain, how equilibrium prices depend on subjective $\pi_i(s)$.

Problem 4 (3p, incomplete markets) Consider exchange economy with 2 consumers and 3 goods: one in date $t = 0$ and two (conditional on realization of state) in date $t = 1$. Endowment is given by $\omega_1 = (2, 2, 0)$ and $\omega_2 = (0, 2, 2)$. Preferences of consumer A over these three goods are given by $\ln(x_0) + \ln(x_1) + \ln(x_2)$ and for consumer B by $2 \ln(x_0) + \ln(x_1) + \ln(x_2)$. Assume $p_0 = 1$. Find prices and allocations of both assets in Radner equilibrium with the three assets structures:

- assume there are two assets with $r_1 = (1, 0)$ and $r_2 = (0, 1)$.
- now assume there are three assets: $r_1 = (1, 0)$, $r_2 = (0, 1)$, $r_3 = (1, 1)$.
- what if there is only one asset with $r_3 = (1, 1)$?

Problem 5 (3p, endogenous incompleteness) Read paper: Competition in Financial Innovation (A. Carvajal, M. Rostek, M. Weretka, Econometrica 80, p. 1895–1936, 2012). Consider two games as analyzed in section 5.1.

- when a Nash equilibrium of a game played between 2 firms offering financial assets leads to a complete market structure?
- explain using intuition resulting from assumptions placed on u''' ,
- how will that equilibrium change if we consider n firms?