Advanced Microeconomics - Problem set 4
Due date: Monday, November 23rd in class

Problem 1 (2pt) *Consider a pure exchange economy with* $u_1(x, y) = x+y$, $u_2(x, y) = \ln(x)+2y$, $\omega_1 = (1, 0)$, $\omega_2 = (0, 1)$. *Find the set of Pareto-optimal allocations and Walrasian equilibria and depict them on the Edgeworth box.*

Problem 2 (2pt) ***First welfare theorem***. *Consider a pure exchange economy with two goods and two consumers. Goods are indivisible and can be only consumed in indivisible quantities (0,1,2, etc). Let both consumers have continuous, strictly monotone and strictly convex preferences.*

(i) Is the Walrasian equilibrium allocation Pareto optimal? Prove or give a counterexample.

(ii) Let now one good be perfectly divisible while the other still not. Is the Walrasian equilibrium allocation Pareto optimal? Prove or give a counterexample.

Problem 3 (2pt) *6.4.4 from our notes.*

Problem 4 (2pt) *6.4.7 from our notes.*

Problem 5 (2pt) *Consider an economy with three commodities* $l = 1, 2, 3$ *and two consumers* $i = 1, 2$. *Preferences are strictly monotone and strictly convex on* $\mathbb{R}^3_{++}$ *(you do not need to prove these properties). For consumer 1 these are represented by utility function*

$$u_1(x_1) = [x_{11}x_{12}(x_{13})^2]^{\frac{1}{4}},$$

for consumer 2 by

$$u_2(x_2) = \frac{1}{-2x_{21}} + \frac{1}{-2x_{22}} + \frac{1}{-x_{23}}.$$

Their endowments are ω_1 *= (1, 0, 1) and* ω_2 *= (1, 2, 1), respectively.*

(a) Do these preferences satisfy LNS on $\mathbb{R}^3_{++}$*?*

(b) Show that the allocation $(x_1, x_2) = ((1, 1, 1), (1, 1, 1))$ *is Pareto optimal.*

(c) Show that any interior Pareto-optimal allocation $(x^o_i)_{i=1,2}$ *satisfies*

$$x^o_{i1} = x^o_{i2} = x^o_{i3}.$$

(d) Next, consider competitive markets. Find the individual demands under price $p = (1, 1, 1)$ *and show that* $p = (1, 1, 1)$ *is not an equilibrium price.*

(e) Find WE.