

Advanced Microeconomics (QEM) - Problem set 1
Due date: Monday, October 12th (end of day) by email

Problem 1 (1p) *Problem 3.B.2 from MWG*

Problem 2 (1p) *Problem 3.B.3 from MWG*

Problem 3 (1p) *Problem 2.4.12 (points 1,2 and 5) from the Lecture Notes.*

Problem 4 (1p) *Problem 2.4.14 from the Lecture Notes.*

Problem 5 (3p) *Consider household with preferences given by:*

$$u(x_1, x_2, \dots, x_T) = \sum_{t=1}^T \beta^{t-1} \ln(x_t),$$

with $\beta \in (0, 1)$ facing prices $p_1, p_2, \dots, p_T$ and income w (that has to be used during the whole T -period lifetime).

- 1. find demand (optimal consumptions path)*
- 2. is consumption increasing, decreasing or constant with time? how does parameter β influence demand x_1 ?*
- 3. find the value function (indirect utility function)*
- 4. write expenditure minimization problem and find Hicksian demand*
- 5. is Hicksian demand increasing, decreasing or constant with time? how does parameter β influence Hicksian demand h_1 ?*
- 6. find the expenditure function*

Problem 6 (5.5p) *Consider the following utility function, called CES (constant elasticity of substitution function):*

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}, \quad \text{where } 0 \neq \rho < 1.$$

This function is obviously strictly increasing. For the above CES function:

- 1. Show that u is strictly quasiconcave.*
- 2. Formulate the Utility Maximization Problem and solve it to derive demand $d_1(p, w)$ and $d_2(p, w)$ [You may assume interior solution and Walras law which follows from strict monotonicity of $u(\cdot)$].*
- 3. Form the indirect utility function $v(p, w)$.*
- 4. Show that the indirect utility function is:*
 - (a) homogeneous of degree zero in (p, w) ,*
 - (b) increasing in w and decreasing in p .*
- 5. Verify the Roy's identity.*
- 6. Formulate the Expenditure Minimization Problem and solve it to derive Hicksian demand $h(p, \underline{u})$ [You may assume interior solution and no excess utility which follows from continuity of $u(\cdot)$]*
- 7. Form the expenditure function $e(p, \underline{u})$.*
- 8. Verify that the expenditure function is concave in p .*
- 9. Verify the Shephard's lemma.*