

TD – Monday, October 1st, 2018

Consumer Theory

The following exercises must be submitted on Monday, October 1st. A particular attention will be given to your presentation.

Exercise 1 (Cobb-Douglas utility function). For all $x = (x_1, x_2) \in \mathbb{R}_+^2$,

$$u(x_1, x_2) = (x_1)^\alpha (x_2)^{1-\alpha} \text{ with } 0 < \alpha < 1$$

1. For every $\bar{x} \in \mathbb{R}_+^2$, determine and draw the indifference curve $I(\bar{x})$ and the upper contour set $U(\bar{x})$.
2. Determine the following properties of u : continuity, differentiability, (strictly) increasing, (strictly) (quasi-)concavity.

Exercise 2. Let $p = (p^1, p^2) \gg 0$ be a price system and $w > 0$ be the wealth of the consumer. Consider the Cobb-Douglas preferences given in the previous TD, Exercise 1.

1. Show that if $x^* = (x^{*1}, x^{*2})$ belongs to the demand of the consumer, then $x^* \gg 0$.
2. Verify that the following utility function represents the Cobb-Douglas preferences on the interior of \mathbb{R}_+^2 :

$$\tilde{u}(x^1, x^2) = \alpha \ln x^1 + (1 - \alpha) \ln x^2$$

3. Determine the following properties of \tilde{u} : differentiability, (strictly) increasing, (strictly) (quasi-)concavity.
4. Determine the demand of the consumer.