

TD – Monday, October 15, 2018

Producer Theory

The following exercises must be submitted on Monday, October 15. A particular attention will be given to your presentation.

Exercise 1. $L = 2$ is the number of commodities. The firm produces commodity y_2 using commodity z as an input. The production function is given by $f(z) = \alpha\sqrt{z}$ with $\alpha > 0$ and $z \geq 0$.

1. Write the transformation function and the profit maximization problem (PMP) of this firm.
2. Show that if $\bar{y} = (\bar{y}_1, \bar{y}_2)$ belongs to the supply of the firm, then $\bar{y}_1 < 0$ and $\bar{y}_2 > 0$.
3. Consider the open and convex set $A = \{y = (-z, y_2) \in \mathbb{R}^2 : z > 0 \text{ and } y_2 > 0\}$. Write the first order conditions associated with (PMP) on the set A , and determine if these conditions are necessary and/or sufficient to solve (PMP) on the set A .
4. Compute the supply and the profit function of the firm.

Exercise 2. Let L be the finite number of commodities. A firm produces commodity L using the other $L - 1$ commodities as inputs. $z := (z_1, \dots, z_l, \dots, z_{L-1}) \in \mathbb{R}_+^{L-1}$ denotes a generic bundle of inputs. Show that if the production function $f : \mathbb{R}_+^{L-1} \rightarrow \mathbb{R}_+$ is concave, then the transformation function defined by

$$t_f(y) := y_L - f(z)$$

is quasi-convex on the convex set $A = \{y = (-z, y_L) \in \mathbb{R}^L : z \geq 0 \text{ and } y_L \geq 0\}$.