

TD – Wednesday, October 3, 2018

Consumer Theory

The following exercises must be submitted on Wednesday, October 3. A particular attention will be given to your presentation.

Exercise 1. Let $(-\infty, \infty) \times \mathbb{R}_+^{L-1}$ be the consumption set. The consumer has strictly convex preferences which are represented by a utility function $u(x) = x_1 + \varphi(x_2, x_3, \dots, x_L)$. We assume $p \gg 0$, and we normalize $p_1 = 1$.

1. Try to show that the demand for commodities $\{2, 3, \dots, L\}$ must be independent of wealth. How does demand for commodity 1 react to changes in wealth w ?
2. Using your previous result, define the indirect utility function as usual, such that $v(p, w) = u(x^*)$, where x^* belongs to the demand, given p and w . Show that $v(p, w)$ is linear in wealth: $v(p, w) = w + \psi(p)$ for some function $\psi : \mathbb{R}_{++}^L \rightarrow \mathbb{R}$ (*You do not need to find ψ*).
3. Now let $L = 2$ and $\varphi(x_2) = \alpha \ln(x_2)$. Solve the UMP as a function of (p, w) (*Recall that we allow demand for commodity 1 to be negative*).

Exercise 2. Let $L = 2$ be the number of commodities. As usual, $x(p_1, p_2, w) = (x_1(p_1, p_2, w), x_2(p_1, p_2, w))$ denotes the demand of the consumer. For every commodity $\ell = 1, 2$, the demand of commodity ℓ is given by

$$x_\ell(p_1, p_2, w) = \frac{w}{p_1 + p_2}$$

1. Prove that this demand is homogeneous of degree zero.
2. Prove that this demand satisfies Walras' Law.
3. State the Weak Axiom of Revealed Preferences (WARP) in the framework of the demand.
4. Prove that this demand satisfies WARP.