General equilibrium - Problem set 1
due date: classes on November, 8th

Problem 1 (2p) *Consider an exchange economy with two consumers and two goods. Find the Pareto-optimal set of allocations and the set of Walrasian equilibria and depict them in the Edgeworth box. Preferences and initial endowments are given by:*

(i) $u_1(x, y) = x + y$, $u_2(x, y) = 2x + y$, $w_1 = w_2 = (1, 1)$,

(ii) $u_1(x, y) = \ln(x) + y$, $u_2(x, y) = \ln(x) + 2y$, $w_1 = (2, 1)$, $w_2 = (1, 2)$.

Problem 2 (2p) *Consider an economy with 3 goods: A, B, C, two firms (b, c) and to consumers $1, 2$: $u_1(x_A, x_B, x_C) = \ln(x_B) + \ln(x_C) = u_2(x_A, x_B, x_C)$ with $\omega_1 = (2, 0, 0)$ and $\omega_2 = (0, 0, 0)$. Firm b produces good B from good A using technology $y_B = 2(-y_{bA})$. Firm c produces good C from A using technology $y_C = (-y_{cA})$.*

- *find a feasible allocation that maximized the sum of utilities.*
- *find a WE with transfers with an allocation computed above with prices of good A normalized to 1.*

Problem 3 (2p) *Proof that, if $\succeq_i$ are rational and strictly convex, and sets X_i convex, then whenever x^*, y^*, p^* is a WE with transfers then x^*, y^* is Pareto optimal.*

Problem 4 (2p) *Consider an exchange economy with two goods and two consumers. Goods are indivisible and can be consumed only in integer numbers. Let both consumers have continuous and strictly monotone and strictly convex preferences.*

(i) It is so, that allocation in any Walrasian equilibrium is Pareto-optimal? Prove of give a counterexample.

(ii) Now, let one of the goods be perfectly divisible and the other not. It is still so, that allocation in any Walrasian equilibrium is Pareto-optimal? Prove of give a counterexample.

Problem 5 (2p) *Consider an exchange economy:* $u_1(x_A, x_B) = \sqrt{x_A x_B}$, $u_2(x_A, x_B) = 2\ln(x_A) + \ln(x_B)$, $e_1 = (1, 0)$, $e_2 = (0, 4)$.

- *Find prices in the Walrasian equilibrium with p_A normalized to 1 and find Lagrange multipliers for budget constraints of each consumer,*
- *Find λ_i, so that the WE allocation (from the previous point) maximizes the social welfare function over the feasible set. Comment referring to Negishi theorem and check if you are right.*

Problem 6 (2p) *15.B.10 from MWG.*