Game Theory - Problem set 1

Due date: classes on March, 19th

Problem 1 (2p) *Consider the Inspection game between worker and principal. Worker can S shirk or W work, while principal inspect I or not NI. Cost of inspection is h, cost of work is g. v is the value of work to the principal. w stands for wage (transfer).*

	I	NI
S	$0, -h$	$w, -w$
W	$w-g, v-w-h$	$w-g, v-w$

- *Assume $w > g > h > 0$.*
- *Find all Nash Equilibria in mixed (and pure) strategies.*

Problem 2 (2p) *Three players $i = 1, 2, 3$ can vote over three alternatives A, B, C. Decisions are taken simultaneously, but none of the players can decide not to vote. The winning alternative is the one that gets most votes. If none of the alternative get most votes then the winner is A. Payoffs depend on the alternative chosen and are the following: $u_1(A) = u_2(B) = u_3(C) = 2$, $u_1(B) = u_2(C) = u_3(A) = 1$, $u_1(C) = u_2(A) = u_3(B) = 0$.*

(i) Write this as a strategic form game;

(ii) Find all pure strategy Nash equilibria.

Problem 3 (2p) *Army A has a single plane, that can be sent to destroy one of the targets. Army B has a single gun, that can be used to protect one of the targets. The value of the target is v_i, where $v_1 > v_2 > v_3 > 0$. Army A can destroy a target only that is not protected by B. Army A's aim is to maximize expected loss of army B, and army B's aim is to minimize such a loss. Write it as a (strictly competitive) strategic form game and find Nash equilibria in mixed strategies.*

Problem 4 (2p) *Consider a first price, sealed-bid auction as a strategic game. Suppose we have $\{1, \ldots, n\}$ players, where i-th player valuation is v_i. Let $v_1 > v_2 > \ldots > v_n > 0$. Each player bets (some nonnegative amount) in a closed envelope and the winner is the one that gives the highest bid. If few players gives the same highest bid then the winner is the one with the smallest i among those that bid the highest. It is a first price auction, i.e. the winner pays the amount he/she bids. Show that in any PS Nash equilibrium player 1 wins.*

Problem 5 (1p) *Diamond introduced the search model, in which player i expands effort $a_i \in [0, 1]$ searching for trading partners, and has a payoff function given by (with $\theta > 0$ being parameter* *characterizing search environment):*

$$u_i(a_i, a_{-i}) = \theta a_i \sum_{j \neq i} a_j - c_i(a_i)$$

Under what conditions placed on c_i, is this a supermodular game? Then, using theorems from the class, what can be said about the NE of this game? How do they depend on parameter θ?

Problem 6 (1p) *In class we considered a Bertrand duopoly model with heterogenous products. You will now analyze Bertrand duopoly model with* homogenous *product. Consider $n = 2$ firms with equal CRS technology and unit costs $c > 0$. Market demand for homogenous product is given by a continuous function d that is strictly decreasing for $p > 0$ and there exists $\bar{p} > 0$ such that $d(p) = 0$ for all $p \geq \bar{p}$. Assume $d(c) > 0$, so that the optimal output level is strictly positive and finite. Then the demand for firm i is given by:*

$$D_i(p_i, p_{-i}) := \begin{cases} d(p_i) & \text{if} \quad p_i < p_{-i} \\ 0.5d(p_i) & \text{if} \quad p_i = p_{-i} \\ 0 & \text{if} \quad p_i > p_{-i} \end{cases}$$

The firms produces to order and so they incure production costs only for output actually sold.

- *show that the unique PSNE is $(p_1^*, p_2^*) = (c, c)$.*
- *carefully draw the graph with the best response correspondence for one firm, e.g. 1.*