Optimization. A first course on mathematics for economists
Problem set 1: Topology

Xavier Martinez-Giralt

Academic Year 2015-2016

1.1 Find the length of the line segment joining $(1, 1, 1)$ to $(3, 2, 0)$.

1.2 For real numbers, prove that

(a) $x \leq |x|, -|x| \leq x$

(b) $|x| \leq a \Leftrightarrow -a \leq x \leq a$, with $a \geq 0$.

(c) $|x + y| \leq |x| + |y|$

1.3 (a) Let $x \geq 0$ be a real number such that for any $\varepsilon > 0, x \leq \varepsilon$. Show that $x = 0$.

(b) Let $S = (0, 1)$. Show that for any $\varepsilon > 0$, there exists $x \in S$ such that $x < \varepsilon, x \neq 0$.

1.4 Let $S = \{(x, y) \in \mathbb{R}^2 | 0 < x < 1\}$ Show that S is open.

1.5 Let $S = \{(x, y) \in \mathbb{R}^2 | 0 < x \leq 1\}$ Is S is open?

1.6 Let $A \subset \mathbb{R}^n$ be open and $B \subset \mathbb{R}^n$.
Define $A + B = \{x + y \in \mathbb{R}^n | x \in A, y \in B\}$. Prove that $A + B$ is open.

1.7 Let $S = \{(x, y) \in \mathbb{R}^2 | 0 < x \leq 1\}$ Find $int(S)$.

1.8 Let $S = \{(x, y) \in \mathbb{R}^2 | 0 < x \leq 1, 0 \leq y \leq 1\}$ Is S closed?

1.9 Let $S = \{x \in \mathbb{R} | x \in [0, 1], x$ is rational$\}$. Find the accumulation points of S.

1.10 Recall the theorem that syas that a set $A \subset \mathbb{R}$ is closed iff all the accumulation points of A belong to A. Verify the theorem for the set
$A = \{(x, y) \in \mathbb{R}^2 | 0 \leq x \leq 1,$ or $x = 2\}$.

1.11 Determine which of the following sets are compact

(a) $\{x \in \mathbb{R} | x \geq 0\}$

(b) $[0,1] \cup [2,3]$

(c) $\{(x,y) \in \mathbb{R}^2 | x^2 + y^2 < 1\}$