

Optimization. A first course on mathematics for economists

Problem set 2: Continuity

Xavier Martinez-Giralt

Academic Year 2015-2016

- 2.1 Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x \sin x$. Show that f is continuous.
- 2.2 Let $f : \mathbb{R} \rightarrow \mathbb{R}^2$ be continuous. Show that $g(x) = f(x^2 + x^3)$ is continuous.
- 2.3 Let $f(x) = \frac{x^2}{1+x}$. Find the points where f is continuous.
- 2.4 Find the sets of points where the following functions are continuous.
- (i) $f(x) = x \sin(x^2)$
 - (ii) $f(x) = \frac{x+x^2}{x^2-1}$, $x^2 \neq 1$, $f(\pm 1) = 0$
 - (iii) $f(x) = \frac{\sin x}{x}$, $x \neq 0$, $f(0) = 1$
- 2.5 Let $A = \{x \in \mathbb{R} \mid \sin x = 0.56\}$. Show that A is a closed set. Is it compact?
- 2.6 Show $f : \mathbb{R} \rightarrow \mathbb{R}$, $x \rightarrow \sqrt{|x|}$ is continuous.
- 2.7 Show $f(x) = \sqrt{x^2 + 1}$ is continuous.
- 2.8 Let $f(x)$ be a cubic polynomial. Argue that f has a real root.
- 2.9 Let $f : [1, 2] \rightarrow [0, 3]$ be a continuous function with $f(1) = 0$, $f(2) = 3$. Show that f has a fixed point in $[1, 2]$.