Optimization. A first course on mathematics for economists
Problem set 4: Classical programming

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4.1 Solve the following problem:

$$\min_{x_1,x_2} 2x_1^2 + x_2^2 \text{ s.t.}$$
$$x_1 + x_2 = 1$$

Give a geometric interpretation to the solution.

4.2 Suppose we have a distribution center that distributes goods to several retail outlets in a city. There are two routes to go from the distribution center to the city A and B. The cost of shipping x units using route A is $ax^2, a > 0$. The cost of shipping y units using route B is $by^2, b > 0$.

(a) Suppose Q units have to be distributed. Determine how they must be allocated to routes A and B to minimize the total shipping cost.

(b) How does the cost change if Q increases by r%?

4.3 An individual has some savings that wants to invest. He wants to minimize risk and obtain an expected return of 12%. There are three mutual funds available yielding expected returns of 10%, 10%, and 15% respectively. Let x, y, and z be the proportion of the savings invested in each of the three funds. The financial experts report that the measure of risk is given by

$$400x^2 + 800y^2 + 200xy + 1600z^2 + 400yz$$

Determine how the individual should distribute his savings among the three funds minimizing the risk.

4.4 An individual has preferences defined over three consumption goods x, y, z. This preferences are represented by means of an utility function

$$U(x, y, z) = 5 \ln x + 8 \ln y + 12 \ln z$$

Unit prices of the goods are $p_1 = 10€$, $p_2 = 15€$, $p_3 = 30€$. The income of the individual is $m = 3000€$.

Find the consumption bundle maximizing the utility of the individual.

4.5 A firm uses three inputs, u, v, w, to produce a certain good. Its production function is

$$Q(u, v, w) = 36u^{1/2}v^{1/3}w^{1/4}$$

The unit prices of the inputs are $p_u = 25€$, $p_v = 20€$, $p_w = 10€$.

(a) Find the levels of the inputs maximizing the output, given that the firm faces a budget constraint of $m = 78000€$

(b) Use the envelope theorem to assess how much can the firm increase the production if its budget increases to 80000€.

4.6 Let $f(x_1, x_2) = x_1 + x_2$. Solve the following problem:

$$\begin{aligned} \min_{x_1,x_2} & x_1 + x_2 \text{ s.t.} \\ & x_1 + 4x_2 = 16 \end{aligned}$$

4.7 Let $f(x_1, x_2, x_3) = x_1x_2x_3, h_1(x, y, z) \equiv x_1^2 + x_2^2 = 1, h_2(x, y, z) \equiv x_1 + x_3 = 1$. Characterize the set of candidate solutions of the following problem:

$$\begin{aligned} \min_{x_1,x_2,x_3} & x_1x_2x_3 \text{ s.t.} \\ & x_1^2 + x_2^2 = 1 \\ & x_1 + x_3 = 1 \end{aligned}$$