Optimization. A first course on mathematics for economists
Problem set 5: Non-linear programming

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5.1 Let $f(x_1, x_2) = -8x_1^2 - 10x_2^2 + 12x_1x_2 - 50x_1 + 80x_2$. Solve the following problem:

$$\begin{aligned} &\max_{x_1,x_2} f(x_1, x_2) \text{ s.t.} \\ &x_1 + x_2 \leq 1 \\ &8x_1^2 + x_2^2 \leq 2 \\ &x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

5.2 Let $f(x_1, x_2) = 4x_1 + 3x_2,\ g(x_1, x_2) = 2x_1 + x_2$ and $x_1, x_2 \geq 0$. Find the candidate solutions to the problem

$$\max_{x_1,x_2} f(x_1, x_2) \text{ s.t } g(x_1, x_2) \leq 10,\ x_1 \geq 0, x_2 \geq 0$$

5.3 Let $f(x_1, x_2) = 2x_1 + 3x_2,\ g(x_1, x_2) = x_1^2 + x_2^2$ and $x_1, x_2 \geq 0$. Find the solutions to the problem

$$\max_{x_1,x_2} f(x_1, x_2) \text{ s.t } g(x_1, x_2) \leq 2,\ x_1 \geq 0, x_2 \geq 0$$

5.4 Solve the following problem

$$\begin{aligned} &\min_{x_1,x_2} x_1^2 - 4x_1 + x_2^2 - 6x_2 \text{ s.t} \\ &x_1 + x_2 \leq 3 \\ &-2x_1 + x_2 \leq 2 \end{aligned}$$

5.5 Let $f(x) = (x-1)^3,\ x \leq 2$ and $x \geq 0$. Show that Kuhn-Tucker first-order conditions are necessary but not sufficient to characterize a maximum of the problem

$$\begin{aligned} &\max_{x_1,x_2} f(x) \text{ s.t} \\ &x \leq 2 \\ &x \geq 0 \end{aligned}$$

5.6 Let $f(x,y) = \frac{1}{x^2+y^2}, g_1(x,y) = y - (x-1)^3, g_2(x,y) = -y, g_3(x,y) = x - 2$, with $g_i(x,y) \leq 0$.

(a) Let S be the set defind by g_1, g_2 and g_3. Provide an argument showing that f has a maximum and a minimum over S.

(b) Show graphically that f has a maximum at $(x,y) = (1,0)$

(c) Verify that the Kuhn-Tucker conditions do not identify that point as a critical point. Explain why.

5.7 Let $U(x,y)$ be a utility function with indifference map represented in figure 1. Let $g(x,y) \leq k$ be the budget constraint. As the figure shows, utility is maximized (given the budget constraint) at the point (x^*, y^*). Show that at that point the indifference curve must be steeper than the budget constraint.

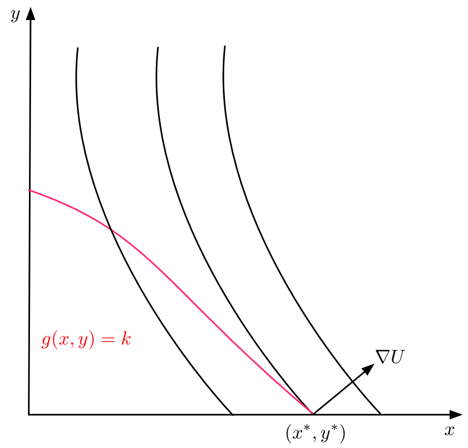

Figure 1: Problem 5.7