Microeconomics 1B – QEM/MMEF | Université Paris 1 Panthéon–Sorbonne
Elena L. del Mercato | 2010-2011

Final Exam (2 hours)

Course Questions.

1. Give the definition of a production economy.

2. State the proposition on the characterization of Pareto optimality in terms of first order conditions. What is the underlying maximization problem under constraints that leads to this characterization?

3. Give the definition of a private ownership economy.

4. State the First Welfare Theorem in a private ownership economy.

Exercise 1. We consider a private ownership economy with two commodities, two consumers and one firm. The individual utility functions are given by

$$u_1(x_1^1, x_1^2) = (x_1^1)^{\frac{1}{2}}(x_1^2)^{\frac{1}{2}} \quad \text{and} \quad u_2(x_2^1, x_2^2) = x_2^1 x_2^2$$

The individual endowments are $e_1 = (\frac{5}{3}, \frac{1}{4})$ and $e_2 = (\frac{1}{3}, \frac{3}{4})$. The firm produces commodity 1 using commodity 2 as an input. The production set of the firm is given by

$$Y = \{(y^1, y^2) \in \mathbb{R}^2 \colon y^1 \le 2\sqrt{-y^2} \text{ and } y^2 \le 0\}$$

The share of consumer 1 is $\theta \in [0, 1]$. Consumer 2 owns the remaining share.

1. Write the definition of competitive equilibrium for this specific private ownership economy.

From now on, the price of commodity 2 is normalized to 1, i.e. $p^2 = 1$.

2. Compute the supply and the optimal profit of the firm.
3. Compute the demand of the consumers.
4. Compute the unique competitive equilibrium with strictly positive consumptions.
5. The equilibrium allocation is Pareto optimal. Why so?

Exercise 2. We consider a production economy with two commodities, two consumers and two firms. The aggregate endowment is given by $(e^1, e^2) = (10, 27)$. The individual utility functions are given by

$$u_1(x_1^1, x_1^2) = 2\ln x_1^1 + \ln x_1^2 \quad \text{and} \quad u_2(x_2^1, x_2^2) = \frac{2}{3}\ln x_2^1 + \frac{1}{3}\ln x_2^2$$

Both firms produce commodity 1 using commodity 2 as an input. The production sets of the firms are given by

$$Y_1 = \{(y_1^1, y_1^2) \in \mathbb{R}^2 \colon y_1^1 \le -\frac{1}{2}y_1^2 \text{ and } y_1^2 \le 0\} \text{ and } Y_2 = \{(y_2^1, y_2^2) \in \mathbb{R}^2 \colon y_2^1 \le \sqrt{-y_2^2} \text{ and } y_2^2 \le 0\}$$

1. Write the definition of feasible allocation for this specific production economy.
2. Write the definition of Pareto optimal allocation for this specific production economy.

3. Using the characterization of Pareto optimality in terms of first order conditions (or the characterization in terms of marginal rates of substitution and marginal rates of transformation), show that any Pareto optimal allocation (x_1, x_2, y_1, y_2) with $(x_1, x_2) \gg 0$ is a feasible allocation satisfying the following equations.

$$\frac{x_1^2}{x_1^1} = 1 = \frac{x_2^2}{x_2^1}, \ \ y_1^1 = -\frac{1}{2} y_1^2, \ \ y_2^2 = -1, \ \ y_2^1 = 1$$

4. Using x_2^2 as a parameter, determine the set of all Pareto optimal allocations (x_1, x_2, y_1, y_2) with $(x_1, x_2) \gg 0$.

Exercise 3. Prove the First Welfare Theorem in a private ownership economy.

Microeconomics 1B – QEM/MMEF
Elena L. del Mercato

Université Paris 1 Panthéon–Sorbonne
2012-2013

Final Exam (2 hours)

Course Questions. Let L be the finite number of commodities.

1. State the proposition which relates the supply of a firm and the derivatives of the cost function.

2. Give the definition of a competitive equilibrium in a private ownership economy.

Consider a pure exchange economy with m consumers.

3. State the proposition on the first order characterization of competitive equilibria.

4. State the proposition on the characterization of Pareto optimal allocations in terms of marginal rates of substitution.

5. State the First Welfare Theorem.

Exercise 1. Let $L = 2$ be the number of commodities. A firm produces commodity 2 using commodity 1 as an input. The cost function of the firm is given by $C(p^1, y_1^2) = 2(y_1^2)^2 p^1$ with $y_1^2 \geq 0$.

Determine the supply $y(p)$ and the optimal profit $\pi(p)$ for any price system $p = (p^1, p^2) \gg 0$.

Exercise 2. We consider a pure exchange economy with $L = 2$ commodities and $m = 2$ consumers. The total initial endowment is $e = (3, 2)$. The consumers have the same preferences represented by $u_i(x_i^1, x_i^2) = x_i^1 + \sqrt{x_i^2}$ for all $i = 1, 2$.

1. Verify that for all $x_1^1 \in]0, 3[$, $\nabla u_1(x_1^1, 1)$ and $\nabla u_2(3 - x_1^1, 1)$ are equal to the vector $(1, \frac{1}{2})$. Deduce that for all $x_1^1 \in]0, 3[$, the allocation $((x_1^1, 1), (3 - x_1^1, 1))$ is a Pareto optimal allocation.

2. Define the price system $p^* = (1, \frac{1}{2})$, and consider the individual initial endowments $e_1 = (2, 0.5)$ and $e_2 = (1, 1.5)$. Determine $x_1^{*1} \in]0, 3[$ such that $p^* \cdot (x_1^{*1}, 1) = p^* \cdot e_1$. Show that $p^* \cdot (3 - x_1^{*1}, 1) = p^* \cdot e_2$.

3. From the previous questions, deduce that $(p^*, (x_1^{*1}, 1), (3 - x_1^{*1}, 1))$ is a competitive equilibrium of the economy defined in question 2.

4. Let $((x_1^1, x_1^2)(x_2^1, x_2^2)) \gg 0$ be a feasible allocation of this economy such that $x_1^2 + x_2^2 = 2$. Show that if $x_1^2 \neq 1$, then this allocation is not a Pareto optimal allocation.

5. Consider different individual initial endowments $(\widetilde{e}_1, \widetilde{e}_2)$ such that $\widetilde{e}_1 + \widetilde{e}_2 = e$. Let $(\widetilde{p}, \widetilde{x}_1, \widetilde{x}_2)$ be a competitive equilibrium for this economy with $(\widetilde{x}_1, \widetilde{x}_2) \gg 0$. Using the First Welfare Theorem and the previous question, show that $\widetilde{x}_1^2 = \widetilde{x}_2^2 = 1$ and $\widetilde{p}$ is proportional to p^*.

Université Paris 1 Panthéon–Sorbonne
Microeconomics 1 – Master M1 MAEF/MMEF/QEM

December 3, 2014 – 11:00 A.M.
Elena L. del Mercato

Exam (1 hour and 40 minutes)

Read and think before you write, and try to be both concise and precise.

Exercise 1 (30 minutes). Let L be the finite number of commodities and let $\mathcal{E} = (u_i, e_i)_{i=1,\dots,m}$ be a pure exchange economy with m consumers.

1. State the theorem on the existence of a competitive equilibrium.
2. State the First Theorem of Welfare Economics.
3. Now, consider a specific pure exchange economy with $L = 2$ commodities and $m = 2$ consumers. The individual utility functions are given by

$$u_1(x_1^1, x_1^2) = x_1^1 \quad \text{and} \quad u_2(x_2^1, x_2^2) = x_2^2$$

$e_1 = (1, 1)$ and $e_2 = (2, 1)$ are the individual initial endowments.

(a) Take a point in the interior of the Edgeworth box, and for both consumers draw the indifference curves and the upper contour sets associated with this point.
(b) Write the definition of a Pareto optimal allocation for this specific economy.
(c) Using the Edgeworth box and the definition of a Pareto optimal allocation, determine geometrically the set of Pareto optimal allocations.
(d) Show that for every consumer $i = 1, 2$, the utility function u_i is quasi-concave and monotonic.
(e) Can we apply the theorem on the existence of a competitive equilibrium? Can we apply the First Theorem of Welfare Economics? Deduce what is the equilibrium allocation.
(f) Using the equilibrium allocation and the properties of a competitive equilibrium, compute the equilibrium price.

Exercise 2 (50 minutes).

1. Let L be the finite number of commodities and let $\mathcal{E} = (Y, (u_i, e_i)_{i=1,\dots,m})$ be a production economy with one firm and m consumers. Assume that the production set Y is represented by a transformation function t. State the proposition on the characterization of Pareto optimal allocations in terms of first order conditions.
2. Now, consider a specific production economy with $L = 3$ commodities, one firm and $m = 2$ consumers. The firm produces two outputs, namely commodities 1 and 2, using commodity 3 as an input. The generic production plan of the firm is denoted by $y = (y^1, y^2, y^3)$ and the production set of the firm is given by

$$Y = \left\{ y = (y^1, y^2, y^3) \in \mathbb{R}^3 \colon y^1 + \beta y^2 \leq 2\sqrt{-y^3} \ \text{ and } y^3 \leq 0 \right\}$$

So, the production set Y is represented by the transformation function

$$t(y^1, y^2, y^3) = y^1 + \beta y^2 - 2\sqrt{-y^3} \text{ with } y^3 \leq 0$$

where the parameter β represents the marginal rate of transformation of output 2 for output 1, we assume that $\beta > 1$.

The two consumers have the same preferences and the same initial endowments. For every consumer $i = 1, 2$, the generic consumption bundle of consumer i is denoted by $x_i = (x_i^1, x_i^2, x_i^3) \in \mathbb{R}^3_+$, the preferences of consumer i are represented by the utility function $u_i(x_i^1, x_i^2, x_i^3) = x_i^1 + \sqrt{x_i^2} + \sqrt{x_i^3}$ and $e_i = (0, 0, 12)$ is his initial endowment.

(a) Determine the set of all Pareto optimal allocations with strictly positive consumption bundles.

(b) Determine the specific Pareto optimal allocation $(\overline{x}_1, \overline{x}_2, \overline{y})$ which guarantees the same consumption in commodity 1 for all the consumers. Compute the supporting price p^* of this specific Pareto optimal allocation $(\overline{x}_1, \overline{x}_2, \overline{y})$ (the price of commodity 1 is normalized to 1).

From now on, we assume that consumer 1 is the only owner of the firm.

(c) Write the definition of a competitive equilibrium for this specific economy.

(d) Consider $(p^*, (\overline{x}_1, \overline{x}_2, \overline{y}))$ determined in question (b). Show that $(p^*, (\overline{x}_1, \overline{x}_2, \overline{y}))$ is not a competitive equilibrium of this economy.

(e) Determine a redistribution of the initial endowments of commodity 3 for which $(p^*, (\overline{x}_1, \overline{x}_2, \overline{y}))$ can be achieved as a competitive equilibrium.

Exercise 3 (20 minutes). Consider a pure exchange economy with $L = 2$ commodities and $m = 2$ consumers. For every consumer $i = 1, 2$, $e_i = (e_i^1, e_i^2) \in \mathbb{R}^2_{++}$ is the initial endowment of consumer i, and the preferences of consumer i are represented by the Log-Linear utility function

$$u_i(x_i^1, x_i^2) = a_i^1 \ln x_i^1 + a_i^2 \ln x_i^2$$

where $a_i = (a_i^1, a_i^2) \gg 0$ is a parameter such that $a_i^1 + a_i^2 = 1$.

1. Quickly verify that u_i is differentiable and concave on $\mathbb{R}^2_{++}$. Compute the individual demand of consumer i at the price $p = (p^1, p^2) \in \mathbb{R}^2_{++}$.

2. We remind that a competitive equilibrium is called a *no-trade* equilibrium if, at equilibrium, for all consumer $i = 1, 2$, the individual demand of consumer i is equal to his initial endowment. Determine the property that the parameters a_i must satisfy at a *no-trade* equilibrium.

Final Exam
Microeconomics 1
QEM 2015/16
Please explain your responses carefully

1. Consider a standard consumer problem with prices $p \in \mathbb{R}^L_{++}$ and wealth $w > 0$. The agent has complete, transitive, and monotone preferences $\succsim$ over bundles in $\mathbb{R}^L_+$.
   (a) State Walras' law and prove that it is satisfied by the demand $x(p, w)$ induced by $\succsim$. Illustrate the idea of your proof in a graph.
   (b) From now on, we consider the following preferences $\succsim$ on $\mathbb{R}^2_+$:
   $$(x_1, x_2) \succsim (y_1, y_2) \Leftrightarrow \min\{x_1, 2x_2\} \geq \min\{y_1, 2y_2\}.$$
   Show that $\succsim$ is monotone but not strictly monotone.
   (c) Determine the demand $x(p, w)$ induced by $\succsim$ for arbitrary $p \in \mathbb{R}^2_{++}$ and $w > 0$.

2. A firm converts a designated input commodity into a single output commodity. As usual, let $z \geq 0$ denote the units of input and let $y \leq f(z)$ denote the level of output, where the production function satisfies
   $$f(z) = \begin{cases} 0 & \text{if } z \in [0, 2], \\ \sqrt{z-2} & \text{otherwise.} \end{cases}$$
   (a) Verify whether or not this technology satisfies nonincreasing returns to scale.
   (b) Normalize to 1 the price of the input and denote by $p > 0$ the price of output. Solve the profit maximization problem and determine the supply of the firm.

3. Consider a group of N individuals. They all have identical preferences over bundles $x_i = (x_{i1}, x_{i2}) \in \mathbb{R}^2_{++}$. For all $i = 1, 2, 3, ..., N$, we have $u_i(x_i) = u(x_i)$, where

$$u(x_i) = \log x_{i1} + \log x_{i2}.$$

Normalize the market price of the first commodity to 1 and denote the relative price of commodity 2 by p. Suppose further that $\sum_{i=1}^{N} e_{i1} = \sum_{i=1}^{N} e_{i2} = \omega > 0$.

(a) State the definition of a competitive equilibrium in this economy.

(b) Determine the price p^* of commodity 2 in a competitive equilibrium.

(c) Pick two particular agents in this economy. Agent 1 with $e_1 = (2, 2)$ and agent 2 with $e_2 = (2, 0)$. What is their equilibrium level of consumption x_1^*, x_2^*?

(d) Suppose that we face a crisis in which the two agents $i = 1, 2$ are no longer trusted by the $N-2$ remaining agents $j = 3, \ldots, N$. The two are from now on excluded from the common market. Instead, agents $i = 1, 2$ have to start their own closed economy where they trade commodities $\{1, 2\}$ with one another, given their endowments $e_1 = (2, 2)$ and $e_2 = (2, 0)$. Determine the competitive equilibrium in this closed economy. Do the two agents agree on whether the crisis is bad or good for them? Comment.

(e) Characterize the set of Pareto optimal allocations, both in the large economy of N agents, $x^O = (x_1^O, ..., x_N^O) \in \mathbb{R}^{2N}_{++}$, and the closed economy with only two agents, $\widehat{x}^O = (\widehat{x}_1^O, \widehat{x}_2^O) \in \mathbb{R}^4_{++}$.

(f) Finally, consider a social planner in the closed economy consisting of agents 1 and 2. She wishes to decentralize a Pareto-optimal allocation in which both agents enjoy the same level of utility. Given initial endowments, determine the transfer t from agent 1 to agent 2 which results in a competitive equilibrium where the utility of agent 1 equals the utility of agent 2.