



# On incentives, temptation and self-control



Łukasz Woźny\*

Warsaw School of Economics, Niepodległości 162, 02-554 Warszawa, Poland

## HIGHLIGHTS

- I consider a principal–agent model with a tempted agent.
- The incentive compatibility constraint is not necessarily binding at the optimal solution.
- The solution to the relaxed problem (without the incentive compatibility constraint) provides a variable pay (even for the separable utility case).
- I show monotone comparative statics results of the optimal contract with the strength of temptation.

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## ABSTRACT

We consider a principal–agent model, where a single agent exhibits problems of self control modeled using Gul and Pesendorfer (2001) type temptation preferences. For a general class of preferences, we characterize the optimal contract in such a setting using standard Grossman and Hart (1983) techniques. Our analysis shows that the incentive compatibility constraint is not necessarily binding at the optimal solution. As a result, the solution to the relaxed problem (without the incentive compatibility constraint) provides a variable pay, which contrasts with the standard results for the separable utility case. These observations result from the fact that in our setting the principal trade-offs incentives and insurance, but also reduction of self control cost for the agent. Our results shed some light on the justification of randomized contracts (see Holmstrom, 1979), the literature on behavioral contracts, but also show that in the presence of strong self-control problems moral hazard cost can be mitigated.

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## 1. Introduction and related literature

Since the seminal work of Strotz (1956) or more recently Laibson (1997), there is now an extensive literature stressing the importance of temptation and self-control problems in explaining individual behavior in economic models.<sup>1</sup> When studying dynamic models with such dynamically inconsistent preferences, theoretical economists have attempted to develop various solutions methods to explain the behavioral observations that have been found in the empirical literature. For example, Strotz (1956) and Caplin and Leahy (2006) used the language of recursive decision theory to compute time consistent solutions. Alternatively, following the contributions of Phelps and Pollak (1968) or Peleg and Yaari (1973) economists have reverted to studying game theoretic

constructions such as Markov perfect equilibria.<sup>2</sup> Since the seminal contribution of Gul and Pesendorfer (2001), however, time consistent (or rational) representations of the models with temptation preferences, or preferences for commitment, have been presented. Their representation allows to solve many technical predicaments usually present, when using models with dynamically inconsistent preferences and, by doing so, allows to extend the analysis of temptation and self-control motives in the otherwise standard models.<sup>3</sup>

Temptation and commitment problems are also present in contractual framework, e.g., within a company between managers and employees, where employees face various temptations for not exerting a desired effort level or delaying a task or a project. Similar considerations arise, when analyzing interactions between companies and clients. Contractual relations are typically modeled in

\* Tel.: +48 22 564 93 22; fax: +48 22 849 53 12.

E-mail address: [lukasz.wozny@sgh.waw.pl](mailto:lukasz.wozny@sgh.waw.pl).

<sup>1</sup> See the work of Angeletos et al. (2001) or Ameriks et al. (2007) for empirical motivation.

<sup>2</sup> For more recent work on game theoretic methods that can be also related to recursive ones, see Balbus et al. (in press).

<sup>3</sup> See also Dekel et al. (2009) and Olszewski (2011) for few extensions of this basic approach.

the literature using principal–agent settings with hidden actions.<sup>4</sup> Some early contributions managed to answer many important questions concerning the shape of optimal contracts (see [Laffont and Martimort, 2002](#) for a survey of results along with references), but still did not allow to rationalize various empirical observations concerning principals’ or agents’ motivations. There is a line of literature, however, that tried to address few behavioral aspects of contracting, like the role of loss-aversion (see [Herweg et al., 2010](#)), fairness (see [Fehr et al., 2007](#)), reciprocity (see [Engelmaier and Leider, 2012](#)), trust (see [Sliwka, 2007](#)) or the role of both extrinsic and intrinsic motives in the short or long run motivation or pro-social behavior (see [Bénabou and Tirole, 2003, 2006](#)) among others. More specifically, for a small selection of the literature on temptation/time consistency in the contractual framework, see [Eliaz and Spiegel, 2006](#), who characterize the optimal contract to screen naive agents with dynamically inconsistent preferences. Additionally, [Heidhues and Koszegi \(2010\)](#) analyze credit markets, when borrowers have a taste for immediate gratification. See also [Della Vigna and Malmendier \(2004\)](#), who characterize the optimal contract design for (partially) naive agents with  $\beta - \delta$  preferences. Finally, we refer the reader to the interesting work of [Esteban and Miyagawa \(e.g., 2006\)](#), who characterize the optimal menu pricing, when consumers face temptation.

Interestingly, moral hazard literature has not studied the problem of optimal incentive schemes in the presence of temptation or self control problems in the general setup. The two interesting exceptions are [Gilpatric \(2008\)](#) and [Yilmaz \(2013\)](#), who characterize the optimal dynamic incentive scheme for naive and sophisticated agents with  $\beta - \delta$  preferences. Specifically, [Gilpatric \(2008\)](#) shows, how the principal can use time-inconsistency of naive agents’ decisions to reduce the optimal cost of implementation, while [Yilmaz \(2013\)](#) shows the optimal dynamic contract for a time-inconsistent but sophisticated agent. In this note we answer the question of optimal, static contract design in the principal–agent model, where preferences of the agent allow for temptation modeled in the spirit of [Gul and Pesendorfer \(2001\)](#), rather than  $\beta - \delta$  setup. Our aim is to keep the model parsimonious, and therefore we focus on a simple, static, single agent–single principal model with two effort levels only. We assume general class of preferences, yet specific family of temptation utilities and we characterize the optimal contract in such a setting using standard [Grossman and Hart \(1983\)](#) techniques. We then generalize our results to the case of multi-action and continuum of actions, as well as general temptation preferences.

Here let us stress that temptation, self-control or commitment problems can arise in contractual framework, even when no moral hazard is present. That is, these two motives can be seen as orthogonal. From this perspective the contribution of the paper is three-fold. First, we characterize the optimal contract for the tempted agent. This allows to understand the incentive schemes that shall be offered to tempted agents with possible applications to over-consumption (or too little savings) problems, or incentive schemes for unemployed. Second, we prove a number of comparative statics results that show, how the optimal incentive scheme depends on a strength of temptation. And third, our model allows for (partial) endogenization of moral hazard cost. Specifically, we propose a model, where the cost of exerting particular action depends on the whole incentive scheme (via the maximal temptation).

More technically, we show a number of things. First, contrary to standard results, the unconstrained problem solution (i.e., solution to the principal’s maximization problem without the incentive compatibility constraint) provides a *variable* pay. Second, in the constrained solution, the incentive compatibility constraint is not

necessarily binding. Both results stem from the fact that in our setting the principal trades off incentives versus insurance, but in the context of the reduction of the agent’s self control cost. Our results shed some light on the theory of motivation, as well as the variable pay schemes but also show that in the presence of strong self-control cost both: solutions to the unconstrained problem and the constrained one coincide, and moral hazard cost is mitigated. Moreover, our model contributes to findings of [Holmstrom \(1979\)](#) and [Gjesdal \(1982\)](#), who managed to show that, if the agent’s utility is separable in actions and wages, then randomized contracts are never optimal. Our results show that reduction of the agent’s self control cost alone can be enough to justify randomized contracts, even if the agent’s utility is separable.

In the next Section 2 we describe our two-action-two-outputs model and assumptions, then characterize the optimal contract in both the unconstrained (relaxed) and constrained problem. In Section 3 we discuss a model allowing for more general temptation utility and more than two action levels. In Section 4 we finish with some discussion and concluding remarks.

## 2. Two actions model

Consider a model with a single principal and a single agent. The principal’s preferences are standard, while the agent’s preferences allow for temptation. [Gul and Pesendorfer \(2001\)](#) show that such preferences can be represented using two utilities  $u, v$ , where  $u$  is a “commitment utility” function, while  $v$  is a “temptation utility” function. In particular, [Gul and Pesendorfer](#) show that the self-control preferences defined over the set of menus with a typical element  $X$  have a representation:

$$V(X) = \max_{x \in X} u(x) + v(x) - \max_{y \in X} v(y).$$

Such preferences defined over menus with the possibility of temptation allow one to rationalize choices of agents exhibiting temptations, yet allow these agents to still possess some ability to exhibit self-control.

To incorporate [Gul and Pesendorfer \(2001\)](#) type of preferences into the principal–agent model, first note that in our setting the menu of (possibly tempting) alternatives is the set of actions available to the agent. That is, even if the agent’s utility depends on actions and wages, he is tempted by items in the action set only. This could be justified, as at the time of the action choice, the contract is already concluded and wages are thus given. Moreover, following the representation and interpretation of [Gul and Pesendorfer \(2001\)](#) preferences (see Section 4 of their paper) we model the agent’s choice in two steps. First, the agent chooses whether to accept a contract or not. If contract is not accepted, he gets a reservation utility<sup>5</sup>  $\bar{u}$ . If, on the other hand, contract is accepted the agent gets some contract  $w$  (possibly dependent on actions or outputs) and his action set is given by  $A = \{a_1, a_2\}$ , where  $a_2$  denotes the costly action. Then, in the second stage, after accepting a contract, the agent makes a choice from  $A$  and evaluate the utility of particular action  $a_j$  using  $u(w, a_j) + v(w, a_j) - \max_{a' \in A} v(w, a')$ . Interpreting,<sup>6</sup> assuming the principal wants to implement action  $a_2$ , the agent can be tempted to choose  $a_1$ . Still,

<sup>5</sup> This could be specified by some constant wage  $\bar{w}$  and a singleton action set  $\bar{A} = \{a_1\}$ .

<sup>6</sup> Such interpretation of [Gul and Pesendorfer \(2001\)](#) model requires a comment. Note, here we do not allow the principal to directly alter the agent’s choice set  $A$  like in [Esteban and Miyagawa \(2006\)](#). Rather, we allow the principal to choose wages that influence the agent’s utility, specifically his cost of self control, and analyze how the optimal wage and action depends on the strength of temptations. This way, the principal affects desirability of a particular action and hence, influences agent’s choice only indirectly.

<sup>4</sup> See [Holmstrom \(1979\)](#) or [Grossman and Hart \(1983\)](#).

note that the most tempting item can depend on the wage scheme offered as both  $u$  and  $v$  are parameterized by  $w$ . Hence, the principal can influence the most tempting item, as well as the cost of self control by selecting from various wage schemes  $w$ .

To link this observation to the Gul and Pesendorfer (2001) paper and its critical set betweenness axiom, note that, if the principal cannot observe the agent's actions and incentive scheme is flat, then the agent's preferences over subsets of the choice sets  $A$  are:  $\{a_1\} \sim \{a_1, a_2\} > \{a_2\}$ . This implies that the agent chooses a low cost (tempting) action, rather than costly  $a_2$ . If the offered wage provides high incentives, then  $\{a_2\} > \{a_1, a_2\} \geq \{a_1\}$  meaning that the agent prefers to choose  $a_2$  (as it shifts the probability of getting the high wage), and although he does not follow temptation, she bears the cost of self control (of not choosing  $a_1$ ). Still this is better than refusing a contract and enjoying  $\{a_1\}$ , as then chances of getting  $w_2$  are lower. Depending on the action that the principal wants to implement (and implied incentive scheme) one of the two presented preference relations will be implied.

We now specify our environment. After accepting an offer, the agent exerts a costly action  $a_j \in A = \{a_1, a_2\}$ , and then production is stochastic with the probability distribution over  $I = 2$  outputs given by  $Q = \{q_1, q_2\}$ . The probability of output  $i$ , when action  $a_j$  is chosen, is denoted by  $\pi_i(a_j) \in (0, 1)$ . The principal does not observe an action, but gets production  $q_i$ , and then rewards the agent with a transfer  $w_i$  (possibly dependent on realized output).

The principal is risk neutral, and her preferences over production and transfers are given by:

$$\sum_{i=1}^2 q_i \pi_i(a_j) - \sum_{i=1}^2 w_i \pi_i(a_j).$$

The agent's commitment utility over rewards and actions is then given by:

$$\sum_{i=1}^2 u(w_i) \pi_i(a_j) - c_{a_j},$$

while his temptation utility is given by:

$$\alpha \left\{ \sum_{i=1}^2 u(w_i) \pi_i(a_j) - \bar{c}_{a_j} \right\}.$$

The costs  $c_{a_j}$  and  $\bar{c}_{a_j}$  denote the commitment and temptation costs of exerting action  $a_j \in A$  respectively, while the utility  $u$  is standard (i.e., is a differentiable, strictly increasing and strictly concave Bernoulli utility function satisfying Inada condition over random rewards). Note, here we assume that commitment and temptation utility differ only in the costs terms, while utility from wage is the same in both. Clearly this is done for simplicity and we comment on possible generalizations with  $v \neq u$  in Section 3. The parameter  $\alpha \in \mathbb{R}_+$  measures the strength of temptation. The agent's reservation utility<sup>7</sup> is  $\bar{u}$ .

We assume that  $q_2 > q_1$ , as well as that action  $a_2$  is more costly than action  $a_1$ , i.e.,  $c_{a_2} > c_{a_1}$  and  $\bar{c}_{a_2} > \bar{c}_{a_1}$ . Without loss of generality,<sup>8</sup> we can set  $c_{a_1} = \bar{c}_{a_1} = 0$ . Also, we assume that the difference in costs satisfies the following:  $\bar{c}_{a_2} > c_{a_2}$ , which reflects the fact that it is harder to motivate the tempted agent. We finally assume that probabilities satisfy the following:  $1 > \pi_2(a_2) > \pi_2(a_1) > 0$ , which implies that high effort shifts probability upward in the

sense of first order stochastic dominance. Also, for the two output levels, this implies that monotone likelihood ratio property is satisfied.

In what follows, we let  $a^t \in A$  denote the most tempting item (generally dependent on the contract offered), i.e., the solution to:

$$\max_{a^t \in A} \sum_{i=1}^2 u(w_i) \pi_i(a^t) - \bar{c}_{a^t}. \quad (1)$$

To sum up, the agent's utility<sup>9</sup> is given by:

$$U(w, a_j) := (1 + \alpha) \sum_{i=1}^2 u(w_i) \pi_i(a_j) - c_{a_j} - \alpha \bar{c}_{a_j} - \alpha \max_{a^t \in A} \left\{ \sum_{i=1}^2 u(w_i) \pi_i(a^t) - \bar{c}_{a^t} \right\}.$$

The term:

$$\begin{aligned} SC(w, a_j) &:= \alpha \left[ \sum_{i=1}^2 u(w_i) \pi_i(a_j) - \bar{c}_{a_j} \right. \\ &\quad \left. - \max_{a^t \in A} \left\{ \sum_{i=1}^2 u(w_i) \pi_i(a^t) - \bar{c}_{a^t} \right\} \right] \\ &= \alpha [u(w_2) - u(w_1)] [\pi_2(a_j) - \pi_2(a^t)] \\ &\quad + \alpha [\bar{c}_{a^t} - \bar{c}_{a_j}] \leq 0 \end{aligned}$$

represents the cost of self control. Trivially,  $SC(w, a_j) = 0$ , if the optimal temptation  $a^t = a_j$ . More importantly, note that cost of self control depends on  $u(w_2) - u(w_1)$ , even though  $a_j$  and  $a^t$  differ (i.e., self control cost can be reduced, even though temptation is still different from the agent's choice).

The goal is to maximize the principal's utility with respect to  $\{a_j, (w_i)_{i=1}^I\}$  subject to the ex-ante participation constraint and incentive compatibility constraint:

$$\begin{aligned} \max_{a_j \in A, (w_i)_{i=1}^I} & \sum_{i=1}^2 (q_i - w_i) \pi_i(a_j), \\ & U(w, a_j) \geq \bar{u}, \\ (\forall a^t \in A) & U(w, a_j) \geq U(w, a^t). \end{aligned}$$

In what follows, we will denote this problem as a constrained one, while if we consider this problem without the incentive compatibility constraint, we will refer to the unconstrained (or relaxed) one. Next note that, when writing the incentive compatibility constraint:

$$\begin{aligned} (1 + \alpha) \sum_{i=1}^2 u(w_i) \pi_i(a_j) - c_{a_j} - \alpha \bar{c}_{a_j} \\ - \alpha \left\{ \sum_{i=1}^2 u(w_i) \pi_i(a^t) - \bar{c}_{a^t} \right\} \\ \geq (1 + \alpha) \sum_{i=1}^2 u(w_i) \pi_i(a^t) - c_{a^t} - \alpha \bar{c}_{a^t} \\ - \alpha \left\{ \sum_{i=1}^2 u(w_i) \pi_i(a^t) - \bar{c}_{a^t} \right\}, \end{aligned}$$

one can subtract temptation term  $\alpha \left\{ \sum_{i=1}^2 u(w_i) \pi_i(a^t) - \bar{c}_{a^t} \right\}$  from both sides of the inequality, so only commitment preferences influence the incentive compatibility constraint. Still note, the incentive compatibility condition depends on  $\alpha$ .

<sup>7</sup> As noted earlier reservation utility can be thought as an outside utility obtained from fixed wage  $\bar{w}$  and action set  $\bar{A} = \{a_1\}$  with  $\bar{u} = u(\bar{w}) - c_{a_1}$ .

<sup>8</sup> As in the standard case ( $\alpha = 0$ ), only the difference between  $c_{a_2}$  and  $c_{a_1}$  affects the incentive compatibility constraint, while cost of the least action can be included into the reservation utility. Hence, we can set  $c_{a_1} = \bar{c}_{a_1} = 0$  and interpret  $c_{a_2}$  as the relative cost of implementing  $a_2$  (versus  $a_1$ ); similarly for  $\bar{c}_{a_2}$ .

<sup>9</sup> Note: as we analyze the choice after accepting an offer, for simplicity we drop from our notation that  $U$  depends on the whole set  $A$ .

We now follow the two step procedure to characterize the optimal choice. First, we determine the minimal cost of implementing each action  $a_j$ , i.e.,  $C(a_1)$  and  $C(a_2)$  respectively; and then we solve:

$$\max_{a_j \in A} \sum_{i=1}^2 q_i \pi_i(a_j) - C(a_j).$$

Observe that as  $a_1$  is the least costly action, the cost of implementing  $a_1$  is simply a flat wage  $\bar{w} := w_1 = w_2$ , such that the participation constraint holds:  $u(\bar{w}) = \bar{u}$ , implying that  $C(a_1) = u^{-1}(\bar{u})$ . In such a case, we have  $SC(\bar{w}, \bar{w}, a_1) = 0$ . Therefore, in what follows, we aim to derive  $C(a_2)$ .

Using standard technique of replacing wages by inverse utilities  $u_i$  to calculate  $C(a_2)$ , we need to solve some strictly convex minimization problem. The presence of self control cost does not change the concavity of the problem (as our constraints are linear in utility values  $u_i$ ). The minimization problem can, therefore, be solved using standard Kuhn–Tucker conditions. That is, by letting  $\lambda$  denote Lagrange multiplier of the participation constraint, and  $\mu$  Lagrange multiplier of the incentive compatibility constraint, the necessary and sufficient condition for the optimal  $w_i$  is:

$$\begin{aligned} & -\pi_i(a_2) + \lambda(1 + \alpha)u'(w_i)\pi_i(a_2) - \lambda\alpha u'(w_i)\pi_i(a^t) \\ & + \mu(1 + \alpha)u'(w_i)[\pi_i(a_2) - \pi_i(a_1)] = 0, \end{aligned}$$

or

$$\frac{1}{u'(w_i)} = \lambda + \lambda\alpha \left[ 1 - \frac{\pi_i(a^t)}{\pi_i(a_2)} \right] + \mu(1 + \alpha) \left[ 1 - \frac{\pi_i(a_1)}{\pi_i(a_2)} \right], \quad (2)$$

or

$$\frac{1}{(1 + \alpha)u'(w_i)} = \lambda \left( 1 - \frac{\alpha}{1 + \alpha} \frac{\pi_i(a^t)}{\pi_i(a_2)} \right) + \mu \left[ 1 - \frac{\pi_i(a_1)}{\pi_i(a_2)} \right].$$

We start with a simple observation.

**Lemma 1.** *The participation constraint is binding in the unconstrained and constrained problems, i.e.,  $\lambda > 0$ .*

**Proof.** Sum Eqs. (2) for all  $i = 1, 2$  to observe that:

$$\lambda = \frac{\pi_1(a_2)}{u'(w_1)} + \frac{\pi_2(a_2)}{u'(w_2)}.$$

Observe that this is true for both  $\mu > 0$  or  $\mu = 0$ . Since  $\pi_i(a_j) > 0$  and  $u$  is strictly increasing, we have that  $\lambda > 0$ . Note also that formula for  $\lambda$  is the same as in the standard case ( $\alpha = 0$ ), although, its value may be different, as the optimal  $w_1, w_2$  depend on  $\alpha$ .  $\square$

Let us now proceed to characterize solutions to the unconstrained (or relaxed problem), i.e., without the incentive compatibility constraint.

### 2.1. Relaxed problem

We first solve the relaxed problem, i.e., our problem without the incentive compatibility constraint. Before doing so, let us stress, that our relaxed (or unconstrained) problem may be different from, so called, “the first best”, for at least few reasons. In our model, in the unconstrained problem, the principal cannot observe the agent’s tempting item  $a^t$ , and hence cannot condition its wage on this element. Hence, there is some loss of efficiency. Next, temptation is an element from  $A = \{a_j\}_j$ , and not  $\{a_j, w_i^{a_j}\}_{i,j}$ . This means that, in the relaxed problem, the most tempting action is determined after the wage scheme was already chosen.<sup>10</sup>

<sup>10</sup> As a result, it is clear there are various ways of understanding the first best solution in our model. We follow our approach to characterize the nature of the constrained solution, by first solving the relaxed and only later the full problem, rather than focusing on the separable observable action case, that is not directly linked to the unobservable action case in our setting.

Having that, we first state few observations that characterize the optimal contract. We want to derive  $C^{NC}(a_2)$ , i.e., the costs of implementing  $a_2$  in the unconstrained problem and for this reason, from now on, we assume that the principal wants to implement action  $a_2$ .

**Lemma 2.** *Suppose the principal wants to implement  $a_2$  with observable actions, then:*

- the most tempting item is action  $a_1$  and
- the optimal contract satisfies:  $w_1^{a_2} \neq w_2^{a_2}$ .

**Proof.** To see the first point assume the contrary, i.e., that temptation is  $a_2$ . Then, by the first order condition (for the case of  $\mu = 0$ ), we have  $w_1 = w_2$ . But in case of a flat wage, the solution to problem (1) is the least costly action (i.e.,  $a_1$ ). This contradicts our supposition.

To see the second point assume the converse, i.e.,  $w_1^{a_2} = w_2^{a_2} =: w^{a_2}$ . Then, as we argued already the solution to problem (1) is  $a_1$ . But this contradicts the first order conditions together with the fixed wage assumption.  $\square$

The previous lemma implies that in order to implement action  $a_2$  the principal must provide a variable pay, even in the relaxed problem. This observation results not from incentives, but because it is cheaper to provide a variable pay and reduce the cost of self control. Now, the first order condition implies:

$$\frac{1}{u'(w_2)} - \frac{1}{u'(w_1)} = \lambda\alpha \left[ \frac{\pi_1(a^t)}{\pi_1(a_2)} - \frac{\pi_2(a^t)}{\pi_2(a_2)} \right], \quad (3)$$

that is positive in our case (by MLRP), if  $a^t \neq a_2$ , implying  $w_2 > w_1$ . Hence, even in the relaxed problem, the optimal contract is monotone. Moreover, the above condition also implies that  $\alpha$  scales deviation from the no-temptation, and hence the full insurance case.

The reason we obtain this result is simple. Notice, the principal observes the agent’s action, but not its temptation. As a result, any contract specifying wage as a function of action (and possibly output  $(w_1^{a_1}, w_2^{a_1}, w_1^{a_2}, w_2^{a_2})$ ) affects choice out of the commitment and temptation utilities, but not necessarily the same way. As a result, in case of a wage that is output independent, temptation remains the same, namely  $a_1$ .

Here observe that result of the lemma states that in the relaxed solution the principal would provide a variable pay but the incentive will be not high enough to change temptation from  $a_1$  to  $a_2$ . That is, there is an upper bound on the difference between both rewards, however.

**Lemma 3.** *The optimal contract  $(w_1, w_2)$  satisfies  $u(w_2) - u(w_1) \leq \frac{\bar{c}_{a_2}}{\pi_2(a_2) - \pi_2(a_1)}$ .*

**Proof.** From Lemma 2 we know that  $a^t = a_1$ . If so, then  $\sum_i u(w_i)\pi_i(a_2) - \bar{c}_{a_2} \leq \sum_i u(w_i)\pi_i(a_1)$ , as otherwise the temptation would be  $a_2$ .  $\square$

Our result for the relaxed case is summarized in the following proposition.

**Proposition 1.** *In the relaxed (unconstrained) problem the principal trade-offs reduction of self control cost versus risk sharing (or providing maximal insurance). The optimal difference between wages is such that  $u(w_2) - u(w_1) \leq \frac{\bar{c}_{a_2}}{\pi_2(a_2) - \pi_2(a_1)}$ .*

Few comments concerning this results. First, it shows that cost minimization leads to variable pay, even though risk sharing calls for a flat wage. To the best of our knowledge, it is a new result in the literature. Second, it is never optimal for the principal (implementing high action) to reduce the agent’s self control cost to zero. Such situation is simply inconsistent with the

principal's maximization. Third, there is an upper bound on the incentives provided by the intrinsic (or self control) motivation. For further references, we denote the optimal unconstrained (relaxed) problem solution  $u(w_2) - u(w_1)$  difference by  $\delta^{NC}$ .

We finish this section with a simple monotone comparative statics result.

**Proposition 2.** *Suppose the principal wants to implement  $a_2$  with observable actions, then the ratio of marginal utilities  $\frac{u'(w_2)}{u'(w_1)}$  is decreasing in the temptation parameter  $\alpha$ .*

**Proof.** To see this observe that the first order conditions imply:

$$\begin{aligned} \frac{u'(w_2)}{u'(w_1)} &= \frac{1 + \alpha \left[ 1 - \frac{1}{\pi_1(a_2)} \right] \left[ 1 - \frac{\pi_2(a^t)}{\pi_2(a_2)} \right]}{1 + \alpha \left[ 1 - \frac{\pi_2(a^t)}{\pi_2(a_2)} \right]} \\ &= 1 - \frac{\alpha \frac{1}{\pi_1(a_2)} \left[ 1 - \frac{\pi_2(a^t)}{\pi_2(a_2)} \right]}{1 + \alpha \left[ 1 - \frac{\pi_2(a^t)}{\pi_2(a_2)} \right]}. \end{aligned} \tag{4}$$

Then, the right hand side is decreasing in  $\alpha$  for  $\pi_2(a_2) > \pi_2(a_1)$ , hence the result follows.  $\square$

### 2.2. Constrained problem

We now turn to the case of unobservable actions and, hence, we bring back the incentive compatibility constraint. Denote  $\phi_\alpha := c_{a_2} + \alpha \bar{c}_{a_2}$ . We start with the following observation:

**Proposition 3.** *Suppose the principal wants to implement  $a_2$ , then:*

- if  $\delta^{NC} \geq \frac{\phi_\alpha}{(1+\alpha)[\pi_2(a_2) - \pi_2(a_1)]}$ , then in the constrained optimal contract  $\mu = 0$ ,
- if  $\delta^{NC} < \frac{\phi_\alpha}{(1+\alpha)[\pi_2(a_2) - \pi_2(a_1)]}$ , then in the constrained optimal contract  $\mu > 0$ ,
- in both cases the temptation is  $a_1$ .

**Proof.** Clearly, if  $\delta^{NC}$  satisfies the incentive compatibility, then in the constrained case  $\mu = 0$ . Alternatively, if  $\delta^{NC}$  does not satisfy the incentive compatibility, then<sup>11</sup>  $\mu > 0$ .

To see the last statement observe that indeed, if  $\mu = 0$  then the result comes from Lemma 2. If  $\mu > 0$ , we know that the difference between rewards is just:

$$\begin{aligned} [u(w_2) - u(w_1)][\pi_2(a_2) - \pi_2(a_1)] &= \frac{c_{a_2}}{1 + \alpha} + \frac{\alpha}{1 + \alpha} \bar{c}_{a_2} \\ &< \bar{c}_{a_2}. \quad \square \end{aligned}$$

This proposition has at least few consequences. First, the possibility of the incentive compatibility that is not binding in the constrained case, is the next striking difference with respect to the no temptation case. In such case moral hazard cost is mitigated by the intrinsic (or self-control) incentives. In fact, we can show that for large  $\alpha$  the incentives provided by the relaxed problem solution are high enough to satisfy the incentive compatibility constraint. Intuitively, the higher the cost of self control, the higher the benefits from reducing it, by offering steep incentive scheme in the relaxed

<sup>11</sup> Indeed, solving the two first order conditions (2) for  $\mu$  we obtain:

$$\mu \frac{\pi_1(a_2) - \pi_1(a_1)}{(1 - \pi_1(a_2))\pi_1(a_2)} = \frac{1 + \alpha \left[ 1 - \frac{\pi_2(a^t)}{\pi_2(a_2)} \right]}{(1 + \alpha)u'(w_1)} - \frac{1 + \alpha \left[ 1 - \frac{\pi_1(a^t)}{\pi_1(a_2)} \right]}{(1 + \alpha)u'(w_2)}.$$

Observe that it is not clear, whether  $\mu > 0$  or  $\mu = 0$ , unless  $\alpha = 0$  or if temptation is  $a_2$ . Although the incentive compatibility implies that  $u'(w_2) \leq u'(w_1)$ , ratios  $\left[ 1 - \frac{\pi_1(a^t)}{\pi_1(a_2)} \right]$  and  $\left[ 1 - \frac{\pi_2(a^t)}{\pi_2(a_2)} \right]$  can be different and hence the incentive compatibility can be binding or not.

problem, and actually the more likely it is to satisfy the incentive compatibility constraint in the constrained problem.

Second and similarly to the relaxed problem, it is never optimal for the principal (implementing high action) to reduce the agent's self control cost to zero. Third, as there is an upper bound on the incentives provided by the intrinsic motivation, sometimes additional incentives are needed to implement high action in the constrained problem.

Using characterization of the optimal contract provided by Proposition 3, we now aim to solve for the constrained optimal contract. To continue and determine the constrained problem cost of implementing  $a_2$ , i.e.,  $C^C(a_2)$  we need to consider two cases, with the incentive compatibility binding, or not. First, assume that  $\mu > 0$ , then:

$$u(w_2) - u(w_1) = \frac{\phi_\alpha}{(1 + \alpha)[\pi_2(a_2) - \pi_2(a_1)]}, \tag{5}$$

where  $\phi_\alpha := c_{a_2} + \alpha \bar{c}_{a_2}$ . Observe that for  $\bar{c}_{a_2} > c_{a_2}$  the difference between  $u(w_2)$  and  $u(w_1)$  is increasing with  $\alpha$ . In the same time the participation constraint gives:

$$\begin{aligned} u(w_2)\pi_2(a_2) + u(w_1)\pi_1(a_2) + \alpha(u(w_2) \\ - u(w_1))(\pi_2(a_2) - \pi_2(a^t)) = \psi_\alpha, \end{aligned}$$

where  $\psi_\alpha = \bar{u} + c_{a_2} + \alpha[\bar{c}_{a_2} - \bar{c}_{a^t}]$ . In such case:  $u(w_1) = \psi_\alpha - \frac{(\pi_2(a_2) + \alpha[\pi_2(a_2) - \pi_2(a^t)])\phi_\alpha}{(1 + \alpha)(\pi_2(a_2) - \pi_2(a_1))}$ , and

$u(w_2) = \psi_\alpha + \frac{(\pi_1(a_2) - \alpha[\pi_2(a_2) - \pi_2(a^t)])\phi_\alpha}{(1 + \alpha)(\pi_2(a_2) - \pi_2(a_1))}$ . From Proposition 3 we know that temptation is  $a_1$ , hence the optimal contract satisfies:

$$u(w_1) = \psi_\alpha - \frac{\pi_2(a_2)\phi_\alpha}{(\pi_2(a_2) - \pi_2(a_1))(1 + \alpha)} - \frac{\alpha\phi_\alpha}{1 + \alpha}, \tag{6}$$

and

$$u(w_2) = \psi_\alpha + \frac{\pi_1(a_2)\phi_\alpha}{(\pi_2(a_2) - \pi_2(a_1))(1 + \alpha)} - \frac{\alpha\phi_\alpha}{1 + \alpha}. \tag{7}$$

Hence, the constrained cost is:

$$\begin{aligned} C^C(a_2) &= \pi_1(a_2)h \left( \psi_\alpha - \frac{\pi_2(a_2)\phi_\alpha}{(\pi_2(a_2) - \pi_2(a_1))(1 + \alpha)} - \frac{\alpha\phi_\alpha}{1 + \alpha} \right) \\ &\quad + \pi_2(a_2)h \left( \psi_\alpha + \frac{\pi_1(a_2)\phi_\alpha}{(\pi_2(a_2) - \pi_2(a_1))(1 + \alpha)} - \frac{\alpha\phi_\alpha}{1 + \alpha} \right), \end{aligned}$$

where  $h(u_i) := u^{-1}(u_i) = w_i$ . Also observe that:

$$\begin{aligned} C^C(a_2) &> h \left( \psi_\alpha - \frac{\alpha\phi_\alpha}{1 + \alpha} \right) = h \left( \bar{u} + c_{a_2} + \frac{\alpha}{1 + \alpha} [\bar{c}_{a_2} - c_{a_2}] \right) \\ &\geq h(\bar{u} + c_{a_2}). \end{aligned}$$

Hence, in such a contract, average transfer is higher than in the  $\alpha = 0$  case, but also the incentive for good (bad) performance is higher (lower) than in the  $\alpha = 0$  case. This is summarized in the next result on monotone comparative statics:

**Proposition 4.** *Suppose  $\mu > 0$ , then the optimal  $w_2$  is increasing with  $\alpha$ , while the optimal  $w_1$  is decreasing in  $\alpha$ .*

**Proof.** First concentrate on  $w_2$ . Observe that in Eq. (7) we have three terms. The middle one is monotone in  $\alpha$  as  $\bar{c}_{a_2} > c_{a_2}$ . Similarly, the sum of the first and the third terms is monotone in  $\alpha$  for  $\bar{c}_{a_2} > c_{a_2}$ .

To see how does  $w_1$  depend on  $\alpha$ , we differentiate the right hand side of Eq. (6) with respect to  $\alpha$  and obtain:

$$\begin{aligned} \frac{1}{(1 + \alpha)^2} (\bar{c}_{a_2} - c_{a_2}) - \frac{\pi_2(a_2)}{\pi_2(a_2) - \pi_2(a_1)} \frac{\bar{c}_{a_2} - c_{a_2}}{(1 + \alpha)^2} \\ = \frac{\bar{c}_{a_2} - c_{a_2}}{(1 + \alpha)^2} \left( 1 - \frac{\pi_2(a_2)}{\pi_2(a_2) - \pi_2(a_1)} \right) \leq 0. \quad \square \end{aligned}$$

In the second case, where the incentive compatibility is not binding, we obtain the relaxed solution with  $C^C(a_2) = C^{NC}(a_1)$ . In such case, there is no distortion between action chosen by the principal in the unconstrained and the constrained solution.

We finish this section with an illustrative example showing graphically possibility of a variable pay in the relaxed problem and not binding incentive compatibility in the constrained case (see Fig. 1). To construct constraints in this figure, recall the participation constraint:

$$u(w_2)[\pi_2(a_2) + \alpha(\pi_2(a_2) - \pi_2(a^t))] \geq \bar{u} + c_{a_2} + \alpha(\bar{c}_{a_2} - \bar{c}_{a^t}) - u(w_1) \times [\pi_1(a_2) + \alpha(\pi_1(a_2) - \pi_1(a^t))],$$

and the incentive compatibility:

$$(u(w_2) - u(w_1))(\pi_2(a_2) - \pi_2(a_1)) \geq \frac{c_{a_2}}{1 + \alpha} + \frac{\alpha}{1 + \alpha} \bar{c}_{a_2}.$$

Next, substitute  $u_2 := u(w_2)$  and  $u_1 := u(w_1)$  to obtain the linear constraints in the  $(u_1, u_2)$  space. The principal's preferences are represented by the indifference curves derived from utility  $\sum_{i=1}^2 \pi_i(a_2)(q_i - h(u_i))$ , where  $h(u_i) := u^{-1}(u_i)$ .

It is interesting to see, how the optimal contract implementing action  $a$  changes with  $\alpha$ . Clearly, for  $\alpha = 0$  the incentive compatibility is binding but as  $\alpha$  increases, the relaxed problem difference  $u(w_2) - u(w_1)$  increases as well. In fact, it will increase continuously as the participation constraint becomes steeper. Clearly, the incentive compatibility constraint increases as well, with constant (45°) slope and with an upper bound at  $(0, \bar{c}_{a_2})$  intersection, though. As a result, at some point the optimal solution will cross the incentive compatibility constraint, and from this point on, the constrained and unconstrained solutions coincide. This is illustrated in Fig. 1, as a move from point Y to Z.

Observe also that for large  $\alpha$  the participation constraint may possess a positive slope in the  $(u_1, u_2)$  space. To see that recall that the participation constraint is given by:

$$u(w_2)[\pi_2(a_2) + \alpha(\pi_2(a_2) - \pi_2(a^t))] = \bar{u} + c_{a_2} + \alpha(\bar{c}_{a_2} - \bar{c}_{a^t}) - u(w_1) \times [\pi_1(a_2) + \alpha(\pi_1(a_2) - \pi_1(a^t))],$$

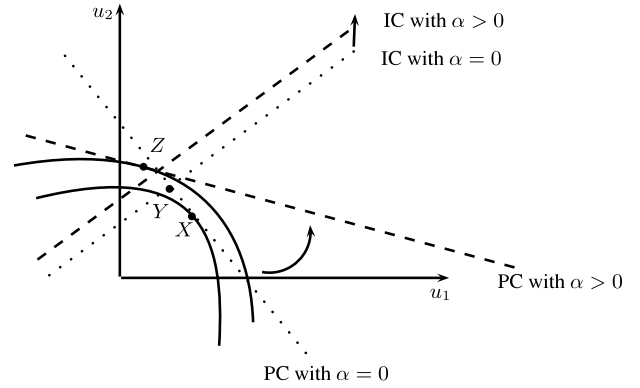
hence, if  $a^t \neq a_2$  and parameters  $\alpha, \pi_2(a_2), \pi_2(a^t)$  are such that  $0 > \pi_1(a_2) + \alpha(\pi_1(a_2) - \pi_1(a^t)) = 1 - \pi_2(a_2) + \alpha(\pi_2(a^t) - \pi_2(a_2))$  the observation follows. To illustrate that refer again to Fig. 1. Increase in  $\alpha$  rotates the participation constraint counter-clockwise. For large  $\alpha$  this can imply a positive slope. In such case, the question of existence of the optimal (unconstrained and constrained) solution may arise as the set of feasible solutions determined by the participation constraint (and the incentive compatibility) is unbounded below. In fact this is not the case, i.e., finite solution always exists. To see that recall that for  $a^t \neq a_2$  one needs  $u(w_2) - u(w_1) \leq \frac{\bar{c}_{a_2}}{\pi_2(a_2) - \pi_2(a^t)}$ . Hence, if the principal wants to go down the participation constraint (with a positive slope), he will encounter the above constraint. Indeed, the slope of the participation constraint is always less than 1 as:

$$-\pi_1(a_2) - \alpha(\pi_1(a_2) - \pi_1(a^t)) = -1 + \pi_2(a_2) + \alpha(\pi_2(a_2) - \pi_2(a^t)) < \pi_2(a_2) + \alpha(\pi_2(a_2) - \pi_2(a^t)),$$

which implies a bounded from below set of feasible solutions given by the participation constraint and constraint implying  $a^t \neq a_2$ . Hence, for the participation constraint with a positive slope, one has the finite solution given by:

$$u(w_1) = \bar{u} + c_{a_2} - \pi_2(a_2) \frac{\bar{c}_{a_2} - \bar{c}_{a^t}}{\pi_2(a_2) - \pi_2(a^t)},$$

$$u(w_2) = \bar{u} + c_{a_2} + \pi_1(a_2) \frac{\bar{c}_{a_2} - \bar{c}_{a^t}}{\pi_2(a_2) - \pi_2(a^t)}.$$



**Fig. 1.** Example of an optimal contract with not-binding incentive compatibility. Dashed lines denote the participation and incentive constraints with positive  $\alpha$ , while dotted lines for  $\alpha = 0$  case. Point X denotes the first best (unconstrained) contract with no temptation, contract Y the optimal second best contract with no temptation, while point Z the optimal contract with  $\alpha > 0$ . Observe that in the unconstrained problem this solution provides a variable pay, such that the incentive compatibility is satisfied but not binding. Additionally, arrows denote, how the participation and incentive constraints move as we shift  $\alpha$ .

This is also a limit of our solution as the strength of temptation approaches infinity and our agent becomes overwhelmingly tempted as in the Strotz model.

### 3. Extensions

#### 3.1. General temptation function

Our model has general commitment preferences but specific temptation preferences. We have chosen this formulation to parameterize the cost of self control. More general, however, the temptation utility  $v$  may be different from commitment one, namely  $u$ . In such case, still the relaxed problem solution will impose a variable pay but the incentive scheme may be different than this given by  $u$  (and derived in the previous section).

Specifically, let us now consider a model with temptation utility given by:

$$\sum_{i=1}^2 v(w_i)\pi_i(a_j) - \bar{c}_{a_j}.$$

With this specification the agent's utility becomes:

$$U(w, a_j) := \sum_{i=1}^2 (u(w_i) + v(w_i))\pi_i(a_j) - c_{a_j} - \bar{c}_{a_j} - \max_{a' \in A} \left\{ \sum_{i=1}^2 v(w_i)\pi_i(a') - \bar{c}_{a'} \right\}.$$

Now assume that  $v$  is strictly convex but such that  $u + v$  is strictly concave. Then, denote  $u_i = u(w_i) + v(w_i)$  and let  $w_i = h(u_i) = (u + v)^{-1}(u_i)$ . With this investment in notation we obtain that:

$$U(u, a_j) := \sum_{i=1}^2 u_i\pi_i(a_j) - c_{a_j} - \bar{c}_{a_j} - \max_{a' \in A} \left\{ \sum_{i=1}^2 v(h(u_i))\pi_i(a') - \bar{c}_{a'} \right\},$$

is strictly concave in  $u$  and its indifference has a convex graph (for a given  $a_j$ ) on the  $(u_2, u_1)$  space with a slope:

$$-\frac{\pi_1(a_2) - \pi_1(a^t)v'(h(u_1))h'(u_1)}{\pi_2(a_2) - \pi_2(a^t)v'(h(u_2))h'(u_2)},$$

where  $a^t$  is some argument solving  $\max_{a^t \in A} \{ \sum_{i=1}^2 v(w_i) \pi_i(a^t) - \bar{c}_{a^t} \}$ .

The set of feasible solutions to the principal's problem is hence convex and the necessary and sufficient conditions for solutions to the cost minimization problem are given by:

$$\frac{1}{u'(w_i) + v'(w_i)} = \lambda \left( 1 - \frac{v'(w_i)}{u'(w_i) + v'(w_i)} \frac{\pi_i(a^t)}{\pi_i(a_2)} \right) + \mu \left[ 1 - \frac{\pi_i(a_1)}{\pi_i(a_2)} \right].$$

Using the arguments as in the proof of [Lemma 1](#), we can show that the participation constraint is binding in both the unconstrained and constrained problems with:

$$\lambda = \frac{\sum_i \frac{\pi_i(a)}{u'(w_i) + v'(w_i)}}{1 - \sum_i \frac{\pi_i(a^t) v'(w_i)}{u'(w_i) + v'(w_i)}}.$$

Similarly to the arguments used in [Lemma 2](#) we can prove that the most tempting item is  $a_1$  and the relaxed problem offers a randomized contract. The incentive compatibility constraint is linear in the  $(u_2, u_1)$  space and the set of feasible solutions in the constrained problem is convex. Again, the relaxed problem difference  $u(w_2) + v(w_2) - u(w_1) - v(w_1)$  can be large enough so that the incentive compatibility constraint of the constrained problem is satisfied but not binding.

### 3.2. More than two actions

In the analysis so far we have assumed there are only two possible actions. This has been done to simplify the analysis but clearly is not without loss of generality. It can be shown that our results extend easily to the more actions case.

Assume now that the set of actions  $A = \{a_1, a_2, \dots, a_n\}$  with  $c_j$  and  $\bar{c}_j$  strictly increasing in  $j$ . Denote by  $\pi_i(a_j)$  the probability of output  $i$ , if action  $j$  is chosen. Assume the principal wants to implement  $a_j$ , then the necessary and sufficient conditions for the cost minimization problem are:

$$\frac{1}{(1 + \alpha)u'(w_i)} = \lambda \left( 1 - \frac{\alpha}{1 + \alpha} \frac{\pi_i(a^t)}{\pi_i(a_j)} \right) + \sum_{k \neq j} \mu_k^j \left[ 1 - \frac{\pi_i(a_k)}{\pi_i(a_j)} \right].$$

Again, if all  $\mu_k^j = 0$ , then the optimal contract is randomized for any  $j > 1$  and the optimal temptation  $a^t = a_k$  satisfies  $k < j$ . To see that assume the opposite, i.e.,  $k \geq j$ . Then the first order conditions yield:

$$\frac{1}{(1 + \alpha)u'(w_2)} - \frac{1}{(1 + \alpha)u'(w_1)} = \lambda \frac{\alpha}{1 + \alpha} \left( \frac{\pi_1(a_k)}{\pi_1(a_j)} - \frac{\pi_2(a_k)}{\pi_2(a_j)} \right),$$

which for  $k \geq j$  (by MLRP) implies  $w_2 \leq w_1$ . But this immediately yields that:

$$0 \geq (u(w_2) - u(w_1))(\pi_2(a_k) - \pi_2(a_j)) \geq \bar{c}_k - \bar{c}_j \geq 0,$$

which means that  $w_2 = w_1$  implying  $k = j = 1$ .

We similarly show that, when implementing action  $a_j$  in the constrained problem, the tempting action is  $a^t = a_k$  with  $k < j$ . To see that assume the opposite, i.e.,  $k \geq j$ , then  $(u(w_2) - u(w_1))(\pi_2(a_k) - \pi_2(a_j)) \geq \bar{c}_k - \bar{c}_j$  and moreover:

$$(u(w_2) - u(w_1))(\pi_2(a_k) - \pi_2(a_j)) > \frac{\alpha}{1 + \alpha} (\bar{c}_k - \bar{c}_j) + \frac{1}{1 + \alpha} (c_k - c_j). \quad (8)$$

Now, the incentive compatibility constraint for output  $k$  yields:

$$(u(w_2) - u(w_1))(\pi_2(a_j) - \pi_2(a_k)) \geq \frac{\alpha}{1 + \alpha} (\bar{c}_j - \bar{c}_k) + \frac{1}{1 + \alpha} (c_j - c_k),$$

which is a contradiction to inequality (8).

To sum up, similarly to our results for  $n = 2$ , indeed the reduced problem (without the incentive compatibility constraints) implies a randomized contract in the multi-action case, and moreover it is possible that (some) incentive compatibility constraints are satisfied but not binding. This includes the case of a local downward incentive constraint satisfied but not binding (see [Grossman and Hart, 1983](#) for assumptions under which this incentive conditions is the only binding one in a model without temptations). A detailed algebraic analysis of such a model is left for further research, however.

### 3.3. Continuum of actions

We now turn to consider a model with two outputs and a continuum of actions  $a \in A := [\underline{a}, \bar{a}] \subset \mathbb{R}$ . For this reason we introduce the following notation:  $c(a)$  is a cost of exerting action  $a$  for the commitment utility, while  $\bar{c}(a)$  for the temptation one. We assume that both  $c, \bar{c} : A \rightarrow \mathbb{R}$  are strictly increasing and strictly convex and twice continuously differentiable. Similarly as above, we let  $\bar{c}'(a) > c'(a)$  for any interior  $a$ . Next, by  $\pi(a)$  we denote the probability of output  $q_2$ , when action  $a$  is taken. We let  $\pi : A \rightarrow [0, 1]$  be increasing, concave and twice continuously differentiable. The principal's problem of minimizing cost of implementing action  $a$  becomes:

$$\begin{aligned} \min_{w_1, w_2} \quad & w_2 \pi(a) + w_1 (1 - \pi(a)), \\ & U(w_1, w_2, a) \geq \bar{u}, \\ & a \in \arg \max_{a' \in A} U(w_1, w_2, a'), \end{aligned}$$

where  $U(w_1, w_2, a)$

$$\begin{aligned} &= (1 + \alpha)[\pi(a)u(w_2) + (1 - \pi(a))u(w_1)] - c(a) - \alpha \bar{c}(a) \\ &\quad - \alpha \max_{a' \in A} [\pi(a')u(w_2) + (1 - \pi(a'))u(w_1) - \bar{c}(a')]. \end{aligned}$$

The necessary and sufficient condition for an interior  $a$  in the incentive compatibility maximization problem is now:

$$u(w_2) = u(w_1) + \frac{\alpha \bar{c}'(a) + c'(a)}{(1 + \alpha)\pi'(a)}, \quad (9)$$

while the one for the optimal, interior tempting action (denoted by  $a^t$ ) is:

$$u(w_2) = u(w_1) + \frac{\bar{c}'(a^t)}{\pi'(a^t)}.$$

As before, when solving the cost minimization problem for interior  $a$  but without the incentive compatibility, one obtains:

$$\frac{1}{(1 + \alpha)u'(w_2)} = \lambda \left( 1 - \frac{\alpha}{1 + \alpha} \frac{\pi(a^t)}{\pi(a)} \right),$$

and similarly for  $w_1$ :

$$\frac{1}{(1 + \alpha)u'(w_1)} = \lambda \left( 1 - \frac{\alpha}{1 + \alpha} \frac{1 - \pi(a^t)}{1 - \pi(a)} \right). \quad (10)$$

Analogously to the reasoning in the two action case, this condition implies the randomized contract in the relaxed problem with  $a^t < a$  (by MLRP).

One obtains similar result in the constrained problem by comparing the two above first order conditions:

$$\frac{\bar{c}'(a^t)}{\pi'(a^t)} = u(w_2) - u(w_1) = \frac{\alpha \bar{c}'(a) + c'(a)}{(1 + \alpha)\pi'(a)},$$

that implies  $c'(a) = \bar{c}'(a^t)$  and as a consequence  $a^t < a$ . Again the relaxed problem difference can be large enough so that the

incentive compatibility constraint is satisfied and moral hazard cost is mitigated. Indeed, the relaxed problem difference in wages is given by (the participation constraint):

$$u(w_2) - u(w_1) = \frac{\bar{u} - u(w_1) + c(a) + \alpha \bar{c}(a) - \alpha \bar{c}(a^t)}{(1 + \alpha)\pi(a) - \alpha\pi(a^t)},$$

with  $w_1$  determined by Eq. (10), which clearly can have a different solution than that of Eq. (9).

#### 4. Concluding remarks

Our study contributes to the analysis of optimal incentives within organizations but also insurance or financial contracts. Specifically, our results indicate that a variable pay may result not only from incentives but also from reduction of the agent's self control cost. Next, if self control problems are intrinsic and high, the moral hazard cost (difference between the unconstrained and the constrained problem solutions) may vanish. Still, for small self control problems additional incentives are needed to motivate agents to choose desired actions. Put it differently, even if incentives motivate agents to choose a desired action, they can be insufficient to reduce the endogenous cost of self-control (i.e., regret of not doing the low cost action).

Our model can be extended to analyze temptation and commitment in the context of contractual framework within a company between managers and employees, where employees face various temptations for not exerting a desired effort or delaying a task or a project. Similar considerations can arise, when analyzing relations between companies and clients on insurance/financial markets with possible applications to the analysis of endogenous default decisions. Also, in specific applications various aspects of temptations and self control can emerge and call for modeling, e.g., random temptations (see Dekel et al., 2009), dynamic temptations (see Noor, 2007) or choice dependent temptation cost (see Olszewski, 2011), bringing some new insights to the behavioral contracts literature.

Holmstrom (1979) showed using Blackwell's informativeness criterion that, for the agent's utility that is separable in actions and wages, randomized contracts are not optimal. Similarly, Gjesdal (1982) presented an example, where for a non-separable utility, randomization is efficient, even if output is a deterministic function of actions. The intuition behind this result is straightforward: the principal can provide additional incentives by making the agent's richer by decreasing his marginal utility of effort. That is, even though it is costly to provide random contract for the risk-averse agent, it can be worth, if it induces higher effort. Our results bring some new arguments to this discussion. In our model the agent's utility is separable but still randomization is optimal in the unconstrained (relaxed) problem. The reason is randomization reduces the agent's self control cost in our model. In fact, for large  $\alpha$  both the principal and the agent can be better off, if randomized contracts are allowed.

Finally, our result share some features of Castro and Yannelis (2011) analysis, who managed to show that the optimal allocation (contract) can be incentive compatible, if agents possess maximin expected utility.

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