

$$x_1(p, w) = \frac{w}{p_2} \quad x_2(p, w) = \frac{w(p_2 - p_1)}{(p_2)^2} \quad \forall p_1 < p_2 \quad \forall w > 0$$

Prove That WARP is NOT satisfied.

W.L.G. Normalize To 1 The price of commodity 2.

$p_2 = p'_2 = 1$, show That $\exists (p_1, w) (p'_1, w')$, $p_1 < 1, p'_1 < 1$ such That (A) $p \cdot x(p', w') \leq w$ and (B) $p' \cdot x(p, w) \leq w'$.

$$(A): p_1 w' + w'(1 - p'_1) \leq w \Leftrightarrow p_1 w' + w' - w' p'_1 \leq w$$

$$\Leftrightarrow w'(p_1 - p'_1) \leq w - w' \Leftrightarrow p_1 - p'_1 \leq \frac{w - w'}{w'}$$

$$(B): p'_1 w + w(1 - p_1) \leq w' \Leftrightarrow p'_1 w + w - w p_1 \leq w'$$

$$w p_1 - p'_1 w - w \geq -w' \Leftrightarrow w(p_1 - p'_1) \geq w - w'$$

$$\Leftrightarrow (p_1 - p'_1) \geq \frac{w - w'}{w}$$

Therefore $\frac{w - w'}{w} \leq p_1 - p'_1 \leq \frac{w - w'}{w'}$ with $x(p, w) \neq x(p', w')$

Take for example $p_1 = p'_1 + \frac{w - w'}{w'}$ with $p'_1 = \frac{1}{4}$

and $\frac{w - w'}{w'} = \frac{1}{4}$ so That $p_1 = \frac{1}{2}$. Then $w' = 4$ and

$w - w' = 1 \Rightarrow w = 4 + 1 = 5$ Finally, notice That

$$\frac{w - w'}{w} = \frac{1}{5} < p_1 - p'_1 = \frac{1}{4}$$

15 minutes