Games and Economic Behavior ••• (••••) •••–•••

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Games and Economic Behavior

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Monotone equilibria in nonatomic supermodular games. A comment ☆

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ARTICLE INFO

Article history:
Received 17 June 2014
Available online xxxx

JEL classification:
C72

Keywords:
Large games
Supermodular games
Games with strategic complementarities
Distributional equilibria

ABSTRACT

Recently Yang and Qi (2013) stated an interesting theorem on the existence of complete lattice of equilibria in a particular class of large nonatomic supermodular games for general action and players spaces. Unfortunately, their result is incorrect. In this note, we detail the nature of the problem with the stated theorem, provide a counterexample, and then correct the result under additional assumptions.

1. Introduction

In an interesting recent paper by Yang and Qi (2013), the authors study the existence of monotone pure-strategy Nash equilibrium in a semi-anonymous nonatomic supermodular game. In contrast to the recent work by Balbus et al. (2014, 2015), where complementarity assumptions for player payoffs are assumed between their individual *actions*, in Yang and Qi (2013) the authors additionally assume complementarities between individual actions and *names* (that could also be interpreted as players' traits). In their paper, the primary technical reason for assuming these additional sources of complementarities is it allows the authors to study Nash equilibria that are monotone in names/traits, which can potentially resolve some problems of measurability of pure-strategies. In particular, the authors assume the name/trait space is a complete chain, and then argue that monotone functions on such a name/trait space are Borel-measurable.[1] If this were true, this would imply that, under pointwise partial order, the space of monotone (and hence measurable) pure-strategies would form a complete lattice, a step that is critical for the application of the Veinott–Zhou's version of Tarski's theorem that the authors use to verify existence of a Nash equilibrium. Unfortunately, monotonicity of pure-strategies in this general setting does *not* imply their measurability.

DOI of original article: http://dx.doi.org/10.1016/j.geb.2013.09.005.

☆ We thank Amanda Fridenberg, Ali Khan, as well as an anonymous referee for very helpful discussions on various issues related to this paper. Łukasz Balbus and Łukasz Woźny also thank the NCN grant No. UMO-2012/07/D/HS4/01393 for financial support during our visit to the Department of Economics, Arizona State University in February 2014. All the usual caveats apply.

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[1] A set T is said to be a *complete chain* if T is (a) a linearly ordered (i.e., T is a partially ordered set such that any two elements are comparable), and (b) complete (i.e., for any subset $T_1 \subset T$, $\sup T_1$ and $\inf T_1$ are both elements of T).

http://dx.doi.org/10.1016/j.geb.2015.05.010

So, in this note, we first show that under the assumptions in Yang and Qi (2013), there is not sufficient structure to verify existence of a Nash equilibrium. We do this by providing a counterexample to a key lemma in the paper needed to prove their main theorem. We then show, how under some additional assumptions, the situation can be remedied. We finally conclude by discussing, how the methods of Balbus et al. (2014, 2015) can be used to study the question of monotone equilibria in a class of large games with more general strategic complementarities (that include nonatomic supermodular games).

2. The environment

We follow the notation in Yang and Qi (2013) in this note. A semi-anonymous game with an ordered set of players' names/traits is a tuple $\Gamma = (T, p, S, f)$, where T is a partially ordered set (a poset) of players' names/traits, $p \in \mathcal{P}(T)$ is a probability distribution on the measurable space $(T, \mathcal{B}(T))$, where $\mathcal{B}(T)$ is the Borel σ-field generated by the interval topology on T, S is an action space of the players, $f : S \times \mathcal{P}(T \times S) \times T \to \mathbb{R}$ is a payoff function where $f(s, r, \theta)$ denotes the payoff for player $\theta \in T$, using action $s \in S$, while facing a joint trait-action distribution given by $r \in \mathcal{P}(T \times S)$.

Denote by $\mathcal{M}(T, S)$ the set of Borel measurable functions from T to S endowed with the pointwise partial order, and let i_T denote the identity mapping. Then, a Nash equilibrium of the game Γ is a function $x \in \mathcal{M}(T, S)$ that satisfies the following:

$$\forall \theta \in T, \forall s \in S \text{ we have } f(x(\theta), p \circ (i_T, x)^{-1}, \theta) \geq f(s, p \circ (i_T, x)^{-1}, \theta), \tag{1}$$

where $p \circ (i_T, x)^{-1}$ is a joint trait-action distribution on $T \times S$ implied by x.

As in Yang and Qi (2013), assume T is a complete chain and S a complete lattice. Endow $\mathcal{P}(T \times S)$ with the first stochastic dominance order. Yang and Qi (2013) impose the following assumptions on game Γ:

Assumption 1. Assume:

- $s \to f(s, r, \theta)$ is order upper semi-continuous for each $\theta \in T$ and $r \in \mathcal{P}(T \times S)$,
- $s \to f(s, r, \theta)$ is supermodular for each $\theta \in T$ and $r \in \mathcal{P}(T \times S)$,
- f has increasing differences with s and (θ, r).

The main existence result of Yang and Qi (2013) states that under Assumption 1, the set of monotone Nash equilibria in $\mathcal{M}(T, S)$ of the game Γ is a nonempty complete lattice. This result is obtained essentially as an application of the Veinott–Zhou fixed point theorem (e.g., see Veinott, 1992 or Zhou, 1994), which in turn is a generalization of the Tarski (1955) fixed point theorem to the case of strong set order ascending best response correspondences. By $I(T, S)$ denote the set of increasing mappings from T to S.

To prove their main theorem, Yang and Qi (2013) claim the following lemma is true:

Lemma 1. *(See Yang and Qi, 2013.) Suppose T is a complete chain and S is a complete lattice, then $I(T, S) \subset \mathcal{M}(T, S)$.*

This lemma is critical for the existence result in Yang and Qi (2013). It claims that *any* monotone increasing function on the complete chain is Borel-*measurable*. In fact, if this lemma was correct, it would imply the set of monotone and measurable functions, $I(T, S) \cap \mathcal{M}(T, S)$, is a complete lattice under pointwise partial orders. Therefore, if the best response correspondence transforms the space $I(T, S) \cap \mathcal{M}(T, S)$ into $I(T, S)$, it necessarily maps into $I(T, S) \cap \mathcal{M}(T, S)$.

Unfortunately, Lemma 1 is not correct, and as a consequence, the main existence theorem in the paper is incorrect as well. Specifically, and related to the issues raised in a recent paper by Balbus et al. (2014), not only is this approach to the proof of the existence in this game generally inappropriate; it is in fact wrong under their assumptions (as the best response correspondence can map *outside* the set of measurable functions).

In the remainder of the paper, we present a counterexample to Lemma 1 (in Theorem 1 in the next section). Then, we discuss sufficient conditions for existence of equilibrium in a version of the actual game considered in Yang and Qi (2013).

3. A counterexample

Consider an interval I of ordinal numbers,[2] and let ω_1 be the first uncountable ordinal number (the least number of the set $\{x \in I : \#([0, x]) \geq \aleph_1\}$). Clearly every initial segment of the subset $[0, \omega_1)$ is countable but $\Omega := [0, \omega_1]$ has cardinality $\aleph_1$.

Following Jech (2010), we define the order topology:

Definition 1. The order topology $\mathcal{O}$ is defined as the topology, whose subbase of closed sets is formed from all sets $F \subset \Omega$ satisfying the following condition:

[2] Our construction in this section is similar to that presented in Theorem 1.14, page 19 and Example 12.9, page 439 in Aliprantis and Border (2006).

$$(\alpha = \sup(F \cap [0,\alpha))) \Rightarrow (\alpha \in F)\,, \tag{2}$$

for all $\alpha \in \Omega$. A closed (open) set in the order topology is said to be order-closed (order-open).

In other words, order-closed sets contain sups of all their subsets. Let $\mathcal{I}$ be the topology generated by the open intervals. We say $\mathcal{I}$ is the interval-topology.

Lemma 2. *Let T be a complete chain and $\mathcal{I}$ its interval topology. Then, $(T,\mathcal{I})$ is a regular and Hausdorff topological space.*

Proof. To show it is a Hausdorff space, let $t_1, t_2 \in T$. As T is a totally ordered set, without loss of generality suppose $t_1 < t_2$. Then, we can separate both points by subsets of

$$\{\tau \in T : \tau < t_2\} = \{\tau \in T : \tau \geq t_2\}^c$$

and

$$\{\tau \in T : \tau > t_1\} = \{\tau \in T : \tau \leq t_1\}^c.$$

We now show that this topology is regular. Let F be a closed subset of T and suppose $t \notin F$. We claim that

$$\underline{t}_F := \sup\{\tau \in F : \tau < t\} < t < \inf\{\tau \in F : \tau > t\} := \bar{t}_F. \tag{3}$$

On the contrary, suppose $t = \underline{t}_F$. Then, there exists a net $(t_d)_{d\in D} \subset F$ such that $t = \sup_{d\in D} t_d$. As F is closed, it is not possible that $t \notin F$. Hence, $\underline{t}_F < t$. Similarly, we prove the second inequality in (3).

Now, we show t and F can be separated by some open sets. Consider four cases:

(i) $\left(\underline{t}_F, t\right) = \left(t, \bar{t}_F\right) = \emptyset$. Then, $\{t\}$ is an open set. Hence, $\{t\}$ and $\{\tau \in T : \tau < t\} \cup \{\tau \in T : \tau > t\}$ are open sets separating t and F.
(ii) $\left(\underline{t}_F, t\right) = \emptyset$ and there is $\alpha \in \left(t, \bar{t}_F\right)$. Hence, $\{\tau \in T : \underline{t}_F < \tau < \alpha\}$ and $\{\tau \in T : \tau < t\} \cup \{\tau \in T : \tau > \alpha\}$ are open sets separating t and F.
(iii) Similarly, we construct open sets separating t and F if $\left(\underline{t}_F, t\right) \neq \emptyset$ $\left(t, \bar{t}_F\right) = \emptyset$.
(iv) Let $\alpha \in \left(\underline{t}_F, t\right)$ and $\beta \in \left(\underline{t}_F, t\right)$. Then, (α,β) and $\{\tau \in T : t < \alpha\} \cup \{\tau \in T : \tau > \beta\}$ separate t and F. □

We now have the following lemma:

Lemma 3. *Topology $\mathcal{O}$ and topology $\mathcal{I}$ are equivalent on Ω.*

Proof. By Lemma 2 $(\Omega,\mathcal{I})$ is a Hausdorff topological space. Hence, by Corollary 3 in Atsumi (1966) $\mathcal{I}$ and $\mathcal{O}$ are equivalent on Ω. □

Definition 2. A set $A \subset \Omega$ is *big* if $A \cup \{\omega_1\}$ includes an uncountable and $\mathcal{I}$-closed set. A set is *small* if its complement is big.

We now prove the following important lemma.

Lemma 4. *There exists a non-Borel set D. That is, there exists $D \notin \sigma(\mathcal{I})$.*

Proof. Observe $\sigma(\mathcal{I})$ is a collection of sets including big and small sets only. To construct a non-Borel set, we know it needs to be neither big nor small. By Theorem 1.8, and Proposition 1.9 in Jech (2010) (see also a theorem by Solovay, 1971) there exists a collection $\mathcal{S}$ which contains uncountably many pairwise disjoint stationary sets[3] in the order topology on $[0,\omega_1)$. By Lemma 3, the order topology is equivalent to the interval topology. Further, observe $[0,\omega_1]$ is just the Alexandroff one-point compactification of $[0,\omega_1)$; hence, if F is closed and unbounded in $[0,\omega_1]$, then the set $F = F_0 \cup \{\omega_1\}$, where F_0 is some closed and unbounded set in $[0,\omega_1)$. Because of this, there is no element from $\mathcal{S}$ that is measurable. On the contrary, suppose we have a big and stationary set $E_1 \in \mathcal{S}$. Then, there is a closed and unbounded set $F_1 \subset E_1 \cup \{\omega_1\}$. Clearly $F \setminus \{\omega_1\}$ is closed on $[0,\omega_1)$. Take some other, say $E_2 \in \mathcal{S}$. Since E_2 is stationary, hence we have $\emptyset \neq E_2 \cap (F_1 \setminus \{\omega_1\}) \subset E_2 \cap E_1$. But this contradicts the pairwise disjoint property of $\mathcal{S}$. Hence, there exists a disjoint and non-measurable set $D := E_1$. □

We now prove the main result of this section.

[3] D is stationary, if for any closed and unbounded set F, $F \cap D \neq \emptyset$.

Please cite this article in press as: Balbus, Ł., et al. Monotone equilibria in nonatomic supermodular games. A comment. Games Econ. Behav. (2015), http://dx.doi.org/10.1016/j.geb.2015.05.010

Theorem 1. *There exists an increasing function $f : \Omega \to \Omega$ that is non-measurable in $\sigma(\mathcal{I})$.*

Proof. From Lemma 4, there is a set $D \subset \Omega$ such that D is not $\sigma(\mathcal{I})$ measurable. Without loss of generality, assume D does not contain ω_1. Define $f(\omega) = \omega + 1$ for $\omega \in [0, \omega_1)$ and $f(\omega_1) = \omega_1$. Define[4] $K := \{x \in \Omega : \exists_{y\in\Omega}\, x = y + 1\}$. Note that since $K = \bigcup_{x\in K}(x - 1, x + 1)$, K is an open set and all the subsets of K are open. Then, $D + 1 := \{x + 1 : x \in D\} \subset K$, and hence it is open. Clearly, f maps into $K \cup \{\omega_1\}$. Then, $f^{-1}(D + 1) = D$. Hence, f is not Borel-measurable. □

4. Discussion and correcting the result

Theorem 1 shows that Lemma 1 in Yang and Qi (2013) is false. What the authors actually show are sufficient conditions under which one has measurability of all isotone functions $f : (X, \le) \to (Y, \le)$, but for Y equipped with the σ-field generated by intervals only. As we show in Lemma 5, the σ-field generated by open (or closed) intervals may be different from the σ-field generated by the topology generated by open intervals.

To illustrate the problem, we state the following lemma:

Lemma 5. $\sigma(\{[0, x) \cup (y, \omega_1], x, y \in \Omega\}) \neq \sigma(\mathcal{I})$.

Proof. Let

$$\mathcal{A} := \{A \subset \Omega : (\#A \le \aleph_0) \vee (\#A^c \le \aleph_0)\}.$$

That is, $\mathcal{A}$ is a set of all countable or co-countable sets. Clearly it is a σ-field. Moreover, each initial segment $[0, x)$ is countable, if $x < \omega_1$, and $[0, \omega_1)$ is co-countable, since its complement is $\{\omega_1\}$. Moreover, $(x, \omega_1]$ is co-countable since its complement is $[0, x]$. As a result

$$\mathcal{A} = \sigma\left(\{A \subset \Omega : (A = [0, x)) \vee (A = (x, \omega_1]) \text{ for some } x \in \Omega\}\right).$$

We will construct an open set outside $\mathcal{A}$. Let $\phi : \Omega \to \Omega$ be defined as follows: $\phi(\omega_1) = 0$ and $\phi(x) = x + 1$ for $x < \omega_1$. Clearly ϕ is a bijection between Ω and $K \cup \{0\}$. As a result Ω has the same cardinality as K, hence K is uncountable. But $K \in \mathcal{I}$. Clearly K^c is closed and contains its least upper bound $\{\omega_1\}$. Hence by Theorem 1.14 and construction on page 19 in Aliprantis and Border (2006), if K^c was countable its least upper bound would be strictly less than ω_1. As a result we conclude K^c is uncountable. And this yields $K \notin \mathcal{A}$. □

There are various ways Lemma 1 (and hence the existence result of Yang and Qi (2013)) can be corrected; namely by adding assumptions to the game Γ, or changing the definition of the game. We discuss these options at this stage.

First, the counterexample is based on the uncountability of the complete chain $\Omega := [0, \omega_1]$. In fact, the statement of Lemma 1 remains valid under the additional assumption that the interval topology on T is *second-countable*. In the case of a countable basis, the σ-field generated by the open (or closed) intervals coincides with that generated by the topology generated by the open intervals. Under this additional condition, then, in fact, every monotone increasing function on such a complete chain will be (Borel) measurable (and, hence, if the best response correspondence transforms the space $I(T, S) \cap \mathcal{M}(T, S)$ into $I(T, S)$, it is also true that it additional maps into $I(T, S) \cap \mathcal{M}(T, S)$, where $I(T, S) \cap \mathcal{M}(T, S)$ is a complete lattice under pointwise partial order). In this case, the existence result follows from a standard application of the Veinott (1992) or the Zhou (1994) fixed point theorems. This remark shows that the strength of the Yang and Qi (2013) theorem is limited to name/trait-spaces that are totally ordered subsets of some compact metric space (see Theorem 2 in Appendix A).

We should mention, this limitation is *significant*, as recently many theoretical and applied papers in the literature of large games have assumed a *saturated* measure space of players names/traits (i.e. a space, whose σ-field is nowhere countably generated). For example, the assumption that players' traits space is saturated is a *necessary* one, if one wants to use the large game in a context of a Bayesian game and apply the Law of Large Numbers (see Sun, 2006).

Second, using different methods, Balbus et al. (2014, 2015) prove the existence of a distributional equilibrium in a class of large games with strategic complementarities using the Markowsky (1976) fixed point theorem.[5] More specifically, under similar assumptions as those in Yang and Qi (2013), they manage to establish conditions under which that set of distributional equilibria possesses the greatest and the least Nash equilibria (but equilibria are *not* necessarily monotone in names/traits).[6] Interestingly, the set of distributional equilibria is *not* a complete lattice in general. This is due to the fact

[4] Let $x \in \Omega$. A successor of x is said to be the least ordinal number in Ω greater than x. This successor is denoted by $x + 1$. A successors set is said to be a set of all ordinals $x \in \Omega$, that are successors of some other ordinals. This set is denoted by K.

[5] Markowsky (1976) presents the following theorem (see Theorem 9 in his paper). Let $F : X \to X$ be increasing, and X a chain complete poset. Then, the set of fixed points of F is a chain complete poset. Moreover, we have $\bigvee\{x : x \le F(x)\}$ is the greatest fixed point, and $\bigwedge\{x : x \ge F(x)\}$ is the least fixed point of F.

[6] That is, neither monotonicity of strategies, nor increasing differences between the action and traits are required in the construction of Balbus et al. (2014). This is important as "monotonicity in names" is often a restrictive form of equilibria to seek.

that in general, the set of probability distributions on $T \times S$ ordered using the first order stochastic dominance in not even a lattice (let alone a complete lattice).

Third, another way to re-establish the result of a complete lattice of Nash equilibria in a large supermodular game is to use the definition of equilibria in a large game due to Schmeidler (1973), where in his definition of Cournot–Nash equilibrium, only almost all players optimize. This construction is also presented in Balbus et al. (2014, 2015), where the authors analyze the set of equivalence classes of measurable functions and then apply the Veinott–Zhou fixed point theorem to a best response correspondence that transforms such equivalence classes of measurable functions. To apply this result to the Yang and Qi (2013) setting, one needs to modify the definition of a Nash equilibrium and require that it is satisfied for a.e. $\theta \in \Theta$. For more results along these lines, but in the context of Bayesian games, see Vives (1990).

Appendix A

In this appendix, we answer to the question: under what properties for name/trait space T will the key lemma in Yang and Qi (2013) be correct? We discuss the case where the name/trait space has an interval topology that is second countable. I turns out, under this additional condition, Lemma 1 will be correct, and their existence result will follow.

Theorem 2. *Suppose T is a complete chain and the interval topology $\mathcal{I}$ is second countable. Then, the topological space $(T, \mathcal{I})$ is a compact metric space.*

Proof. By Lemma 2 $(T, \mathcal{I})$ is a regular Hausdorff topological space. Hence, by Urysohn Metrization Theorem (Theorem 3.40 in Aliprantis and Border, 2006) it is a separable metric space. Now, we show $(T, \mathcal{I})$ is compact. T is a complete chain, hence a complete lattice. By Theorem 20 in Birkhoff (1967), $(T, \mathcal{I})$ is compact. □

We finish by showing that the order topology defined by Jech (2010) is equivalent to that defined[7] by Atsumi (1966): i.e. $x_j \to x$ if and only if:

$$x = \bigvee_i \bigwedge_{j \geq i} x_j = \bigwedge_i \bigvee_{j \geq i} x_j. \tag{4}$$

To see this equivalence, consider the next lemma:

Lemma 6. *On the set Ω the order topology defined by Jech (2010) is equivalent to that defined by Atsumi (1966).*

Proof. Let $x_i \to x$ according to Jech. Then, if $y < x$, then $y < x_i \leq x$ eventually in i. Then, for sufficiently large i:

$$y < \bigwedge_{j \geq i} x_j \leq \bigvee_{j \geq i} x_j \leq x$$

eventually in i. As $\bigwedge_{j \geq i} x_j$ is increasing and $\bigvee_{j \geq i} x_j$ is decreasing, we have:

$$y \leq \bigvee_i \bigwedge_{j \geq i} x_j \leq \bigwedge_i \bigvee_{j \geq i} x_j \leq x.$$

As $y < x$ is arbitrary, hence:

$$x = \bigvee_i \bigwedge_{j \geq i} x_j = \bigwedge_i \bigvee_{j \geq i} x_j.$$

This implies convergence as defined by Atsumi.

Conversely, let $y < x$, and assume (4) holds. Then,

$$y < \bigwedge_{j \geq i} x_j \leq x$$

eventually in i. Hence,

$$y < \bigwedge_{j \geq i} x_j \leq x_i \leq x$$

eventually and this implies convergence according to Jech. □

[7] The order topology defined in Jech (2010) is also equivalent to the order topology defined in Gierz et al. (2003, page 217) applied for directed complete posets. Indeed, since Ω is a totally ordered set, all subsets include their minima.

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References

Aliprantis, C.D., Border, K.C., 2006. Infinite Dimensional Analysis. A Hitchhiker's Guide, 3rd edn. Springer Verlag, Heidelberg.
Atsumi, K., 1966. On complete lattices having the Hausdorff interval topology. Proc. Amer. Math. Soc. 17 (1), 197–199.
Balbus, Ł., Dziewulski, P., Reffett, K., Woźny, Ł., 2014. A qualitative theory of large games with strategic complementarities. Mimeo.
Balbus, Ł., Dziewulski, P., Reffett, K., Woźny, Ł., 2015. Differential information in large games with strategic complementarities. Econ. Theory 59 (1), 201–243.
Birkhoff, G., 1967. Lattice Theory, 3rd edn. American Mathematical Society Colloquium Publications, vol. 25. Amer. Math. Soc., Providence, RI.
Gierz, G., Hofmann, K.H., Keimel, K., Lawson, J.D., Mislove, M., Scott, D.S., 2003. Continuous Lattices and Domains. Encyclopedia of Mathematics and Its Applications, vol. 93. Cambridge University Press.
Jech, T., 2010. Stationary sets. In: Foreman, M., Kanamori, A. (Eds.), Handbook of Set Theory. Springer.
Markowsky, G., 1976. Chain-complete posets and directed sets with applications. Algebra Universitae 6, 53–68.
Schmeidler, D., 1973. Equilibrium points of nonatomic games. J. Stat. Phys. 17 (4), 295–300.
Solovay, R., 1971. Real-valued measurable cardinals. In: Scott, D. (Ed.), Axiomatic Set Theory. Amer. Math. Soc., Providence, RI.
Sun, Y., 2006. The exact law of large numbers via Fubini extension and characterization of insurable risks. J. Econ. Theory 126 (1), 31–69.
Tarski, A., 1955. A lattice-theoretical fixpoint theorem and its applications. Pacific J. Math. 5, 285–309.
Veinott, 1992. Lattice programming: qualitative optimization and equilibria. MS Standford.
Vives, X., 1990. Nash equilibrium with strategic complementarities. J. Math. Econ. 19, 305–321.
Yang, J., Qi, X., 2013. The nonatomic supermodular game. Games Econ. Behav. 82 (C), 609–620.
Zhou, L., 1994. The set of Nash equilibria of a supermodular game is a complete lattice. Games Econ. Behav. 7, 295–300.

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