Erasmus Mundus Joint Master Degree (EMJMD)
QEM1 (First Year) - 2021/2022

Last Name: ______________________ **First Name:** ______________________

ID Number: ______________________ **Program:** ______________________

OPTIMIZATION – Final Exam

December 9th, 2021

Time allowed: 120 minutes.

- This paper contains 3 exercises.
- Try to be as neat and rigorous as possible, justify any step (if you refer to theorems or properties, quote them). It is required that your solutions be well legible and well organized.
- All computations have to be explicitly shown.
- The exam is closed book.

Exercise 1. *(30 minutes)*

Given the function

$$f(x,y,z) = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} + xyz$$

(a) write the domain of f;

(b) write the first order optimality conditions and find the candidate points in order to optimize f;

(c) classify candidate points, carefully showing the criteria you are using;

(d) does the function f have a global maximum or minimum? Justify your answers.

Exercise 2. *(40 minutes)*

Given the constrained optimization problem

$$\max f(x,y) = 6y + 2xy - 2x^2 - 2y^2$$

under the contraints

$$\begin{array}{lcrcl} 2x & + & y & \leq & 2 \\ x^2 & - & y & \leq & 1 \end{array}$$

(a) say if the problem admits a solution (*hint*: give a geometric representation of the feasible region);

(b) study the constraint qualification conditions;

(c) write the first order optimality conditions;

(d) solve the auxiliary problem

$$\max f(x,y) = 6y + 2xy - 2x^2 - 2y^2$$

under the unique constraint

$$\begin{array}{lcrcl} 2x & + & y & \leq & 2 \end{array}$$

and show that the solution is also a solution of the initial problem with the two constraints.

(e) estimate the approximate change in the maximum value when the constraints are replaced by

$$\begin{array}{lcrcl} 2x & + & y & \leq & 2.03 \\ x^2 & - & y & \leq & 0.98 \end{array}$$

Exercise 3. *(50 minutes)*

A consumer faces a T-period planning horizon, where T is a finite positive integer. Her initial wealth is $w_0 \in \mathbb{R}_+$ and, if she begins a period with a wealth of w and she consumes c in that period, then at the beginning of the next period her wealth is $(w-c)k$ where $k = 1 + r$ and $r \geq 0$ is the interest rate. In each period, the utility of the consumption c is given by $u(c)$.
At all times t, consumption c_t and wealth w_t must be non-negative.
The consumer's objective is to maximize the total utility over the T-period horizon.

(a) By writing the objective function and all the contraints, formalize the *Consumers multi-period utility maximization problem* when it is assumed

$$u(c) = \sqrt[3]{c} \quad \text{and} \quad T = 2;$$

(b) write the solution path $(w_0, c_0), (w_1, c_1), (w_2, c_2)$;

(c) compute the maximum value $V_0(w_0)$;

(d) with reference to the new problem

$$\max \sum_{t=0}^{T} \sqrt[3]{c_t}$$

under the same assumptions on w_t and c_t, identify the generalizations of the solutions proposed above, that is c_t and $V_0(w_0)$. Proof by induction is not required.
