Optimization — QEM1
Midterm Exam 2019/2020 (1h)

Exercise 1

For each of the following fonctions, determine the critical points and their nature:
a) $f(x, y) = x^4 + y^4 - 2(x - y)^2$, for all $x, y \in \mathbb{R}$,
b) $g(x, y) = x[\log(x)^2 + y^2]$, for $x > 0$ and all $y \in \mathbb{R}$.

Exercise 2

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = y^5 - 4y^4 + 4xy^2 - x^2$ for all $x, y \in \mathbb{R}$.
2.1) Compute the partial deriviatives of f at every point in $\mathbb{R}^2$.
2.2) Show that there exists a real-valued function φ, defined on some interval I containing 1 such that $f(x, \varphi(x) = 0$ for every $x \in I$ and $\varphi(1) = 1$.
2.3) Show that φ has a tangent at 1 and give its equation.

Exercise 3

3.1 Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ assumed to be Fréchet-differentiable. Show that f is convex iff it satisfies $f(y) \geq f(x) + f'(x) \cdot (y - x)$ for all $x, y \in \mathbb{R}^2$.

3.2 Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is convex and Fréchet-differentiable. Show that $x \in \mathbb{R}^2$ is a global minimum iff it is a critical point.